

## A NEW APPLICATION FOR RECTIFYING METRIC PHOTOGRAPHS USING GRAPHIC METHODS.

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### KEY WORDS

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### ABSTRACT

The current computer applications of vectorial drawing and treatment of images allows to get rectified fotograms applying the geometric fundamentals that base this type of transformations in a way relatively simple. The rectification of fotograms has its geometric nature in the product of homologies in the space, as demostrated Scheimplüg developing its mechanic rectifier. Now, by means of the employment of computer techniques like rendering of images is possible to resolve this type of transformations with a graphic establishing and considerable accuracy, but without employing necessarily specific software that offer different providers based in numerical calculation of coordinates. Besides illustrating the projective nature of the question, that should must be studied to understand fundamentals of analytic methods for photogrammetric survey, we will also show how it result rather simple to apply these graphic gears for the rectification of façades and other problems that may occur in the surveying of buildings.

The rectification of photograms results every time more interesting for professionals of the photogrammetry in the realization of certain works of architectural survey. So, when the model photo can be photographed more or less vertically and it is relatively plane, to take of a complete photograph- or several partials- and rectificate it in a setting scale, proportions us a valuable document of work, that contains the real information, without passing through the abstract filter of linear interpretation, to know geometries, lesions, deteriorations, etc., of the object, that can also be measured with enough approach. Such is the case of numerous façades and interior elevations of historic buildings in those that arrange of rectified photographs results today an indispensable documentation of base for any intervention or cataloguing.

The origin of this technique comes from the aerial photogrammetry destined to cartographic surveys, and its development has been conditioned until recently due to the limitations of the analogical transformers based on mechanic geniuses for its operation. However, at the present time the enormous development of computer sciences applicated to photogrammetry has facilitated many its employment. The techniques of straightening digital images have gotten that the rectification of a photogram is currently only a little over that an option of a computer application in what we only must select the coordinates of a series of points that define the plane basis of the model.

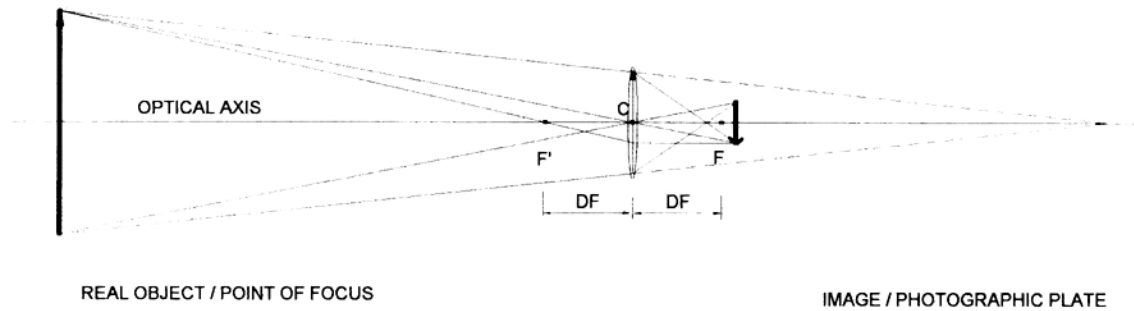
This comparative simplicity has permitted that the technique of rectification of photograms results particularly useful also in works of architectural survey. However the enormous capacity of resolution that offers the computer sciences now, could not make us forget the geometrics fundamentals that based the first

photographic rectifiers and that are the same fundamentals of the analytic methods of calculation that employ the current informatic applications, because in otherwise it will be the own reliability of the results that we get with this potent gears, the one what could come to remain in interdict

It is surprising the scarce attention that is lended to this question in the majority of the treatise about photogrammetry, for us it is an error because the geometrics fundamentals, particularly the projectives, constitute a basic support of photogrammetry, the same as the other branches of the mathematics like algebra, infinitesimal calculation or statistics. However in difference to these, the geometry imposes a graphic language to develop a logical peculiar discourse, what involves a series of difficulties from the beginning in order to advance in its study and comprehension, that in our opinion are in the basis of the current progressive abandon of the interest for this question.

### Projective generation.

The importance of this graphic discourse is due precisely that it proportions a scientific explanation to the fact of graphic projections, separating it in different simple transformations. In fact supposes a fundamental for the operation of represent in cylindric system a material reality that we have perceived before in conic system of projections. Let us see a small approach to these geometrics fundamentals, in which is based the straighten of photograms, like a part of a wider context in that we want to enhace the importance of geometry like latest and principal fundamental of the different techniques of representation.



<b>SYST. INFORMATIC</b>	<b>POINT DEST.</b>	← DIS →	<b>CAMERA</b>	← ZOOM →	<b>SCREEN</b>
<b>SYST. OPTIC. / PHOTOGRAPH.</b>	<b>POINT. FOCUS</b>	← NO →	<b>OPTICAL CENTER OBJECT LENS</b>	← FOCAL →	<b>IMAGE</b>
<b>SYST. GEOMETRIC</b>	<b>(MAIN POINT)</b>	← NO →	<b>POINT OF VIEW</b>	← NO →	<b>PLANE OF REPRESENTATION</b>

Figure 1: Relationships between the systems informatic, optic-photographic and geometric in a general model of computerized transformation of photograms. See the vertical correspondence between them, and how the informatic system is more complete than the other because incorporates the gear called "dis" that permits even to move camera position through main distance line. Besides have no problem with focus when rendering digital images.

The greater difficulty in order to learn projective geometry is its enormous grade of abstraction, particularly in the first phase of definition of its language appropriated call geometric generation. To begin with, we have to introduce the projective concept of projection that is defined from a graphic point of view as the ensemble of projective rays that can be established between an origin and an object to project. In general terms both of them could be a point, straight line or plane, as they can be considered graphic elementary entities, which generate several classes of ensembles of rays named bundles and also geometric shapes. At the same time origin and object can be reals or imaginaries points, what introduce the difficulty that supposes from the learner to elaborate an graphic discourse with these elements named of the infinite.

When both origin and object are points, we generated the simplest type of projective bundle that is called rectilinear series of points: it is formed by the infinites points aligned with object and origin, so that each point of the series while graphic entity is uni-dimensional.

This bundle represents the graphic side of an algebraic entity called projective space of dimension one, that comes to be the ensemble of the quotients formed by the classes of equivalence of the elements of the vectorial space  $R^2$  of the reals numbers of dimension two, to those that has applied a special relationship of equivalence called colinearity, by virtue of what two element are related if the coordinates of one can be expresed like a linear combination of the other.

The dimension of the vectorial space allows to give homogeneous coordinates to the imaginary elements by

means of a determined agreement. In our case these are the imaginary points of each rectilinear series as they are generated by a special uni-dimensional colinearity. It justify that the imaginary elements in any projectivity - relationship biunanimous between object and bundle-behaves exactly like the other reals, what in graphic terms means the performance of the classic axiomatic basic of incidence.

Although the projective branch has born from the graphic side above all<sup>1</sup>, in the majority of the current texts it's not emphasized how the origin of the graphic and analytic methods of representation is the same, and therefore every graphic property can be expressed faithfully also in an analytical maner, capable of therefore an informatic treatment. Here is the enormous importance of fathoming the geometric origin of the majority of tools that proportion us the applied computer sciences of representation in order to employ its enormous possibilities with all rigor for generate good results.

Any question of graphic representation implicates some determined relationships of projectivity between shapes of first category at fewer, but we cannot extend more in this brief summary. Determinate them could be the first pass in order to apply more precise informatic treatment, particularly when the model presents special complexity. If we consider a rectilinear series of points like object to project, and a real or imaginary point like origin, we also generate a first category shape called straight line bundle: each straight line of the bundle will be formed by a rectilinear series.

<sup>1</sup> Specially departing of metric definitions given by Staudt, Chasles or Poncelet.

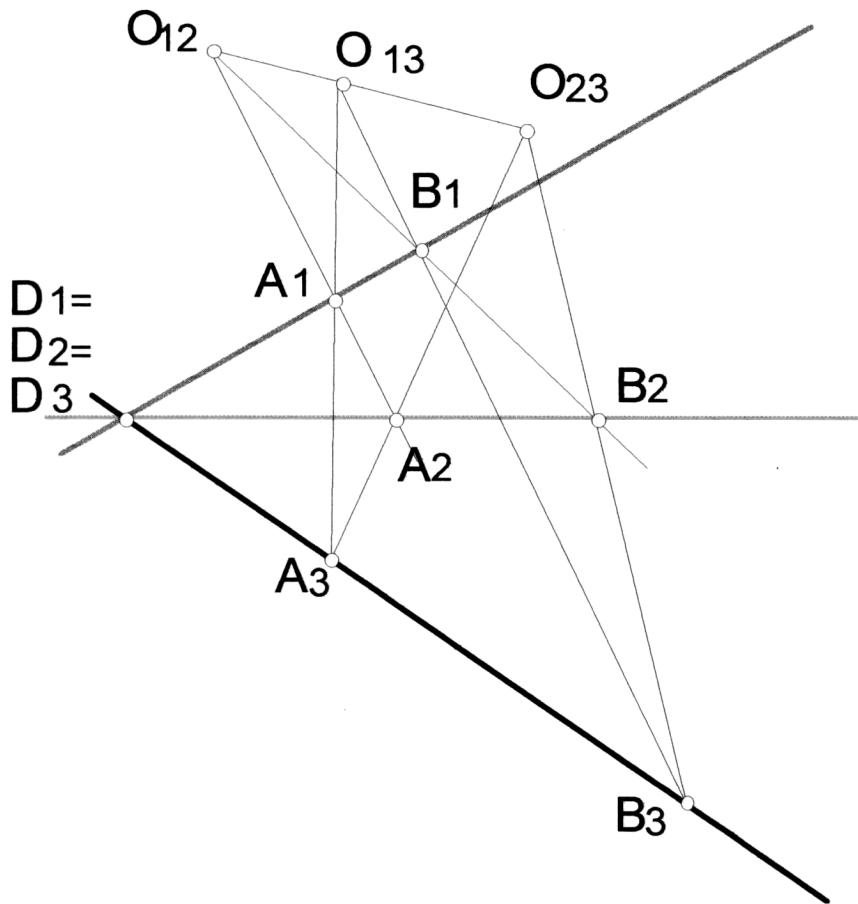


Figure 2: Product of projectivities between shapes of first category.

If the bundle is intersected for a new rectilinear series, what supposes a precise operation of matrix calculation, could be established a biunanimous relationship between each point of the series that intersects the bundle, each ray of the bundle, and each point of the series that generates it. This is a perspectivity, by virtue of what each point of the first represents faithfully to each point of the last.

This model explains the projective, and therefore mathematical, fundamental of any mechanism of graphic representation. If we have a photogram and suppose a section of the model by a plane that contains the optic center of the camera, we could identify easily the elements of this perspectivity between shapes of first category. The face of the model is origin/object of the rectilinear series, the object lens of the camera is the object/origin of the bundle, and its intersection with the photographic film plane represents the section of the bundle that generates the biunanimous relationship of perspectivity, and therefore the image printed in the film represents reliably the original shape.

This projective analysis is completed taking account the perspectivities generated between shapes of the second category. In graphic terms they are defined by bundles with origin/object in a real/imaginary point and

object/origin in the points/straights lines that belong to a plane, and therefore have bi-dimensional character.

The representation of a plane face of an object can be understood as a perspectivity between second category bundles, called customarily spatial relations, for establish following the graphic/analytic transformations needed to a plane representation as first category shapes.

If the linear series that intersect a first category bundle is considered again as an object to project from a second point origin, and we return to intersect this second bundle by means of a third series, then the first and the last series are also related perspectively by means of a third bundle generated starting from a third point origin that results necessarily aligned between the other two.

This operation named product of projectivities, and its precise algebraic correlation, permit a first approach to the geometric fundamental of the rectification of photograms. In the previous assumption, the first linear series is the face of the model, the second the photographic plate and the third the section of the plate re-photographed. (Figure 2)

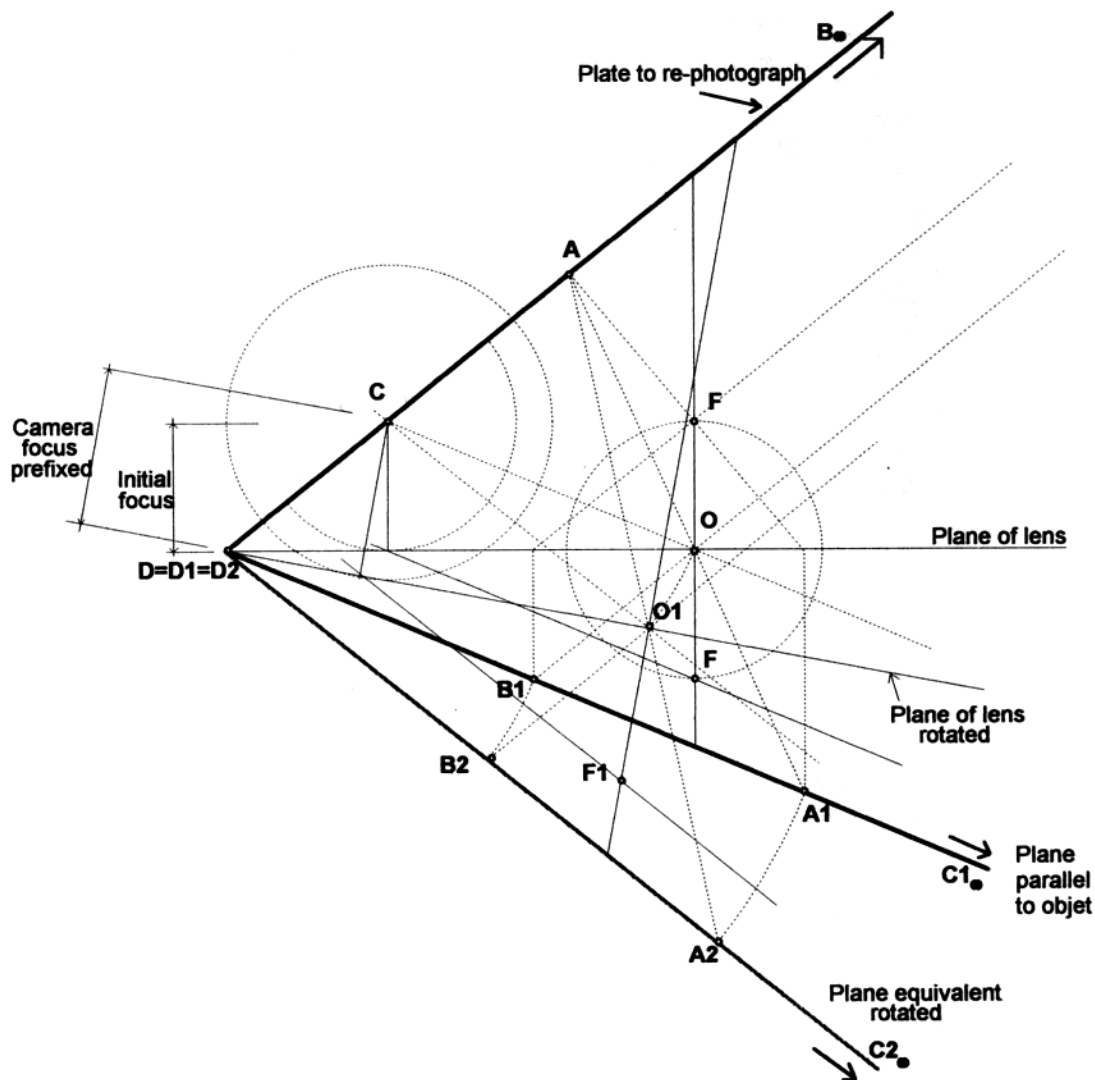


Figure 3: Position of Scheimpflug of the three perspective series representative of the rectification of a planar face of a model. Bundle with center  $O$  between plate to re-photograph and parallel plane to the object (rectified image) but with initial focus. Bundle with center  $O_1$  between plate to re-photograph and plane equivalent to the rectification but rotated so that is obtained with the camera focus prefixed.

### Mechanic Rectifiers.

The operations with products of linear series base the rectification of photograms. From a geometric point of view, the relationships between linear series can admit variables like real or imaginary origin, and common or different origin between one and another. Specific configurations of these projective systems suppose concrete questions of representation.

So, in base to the properties of these products is possible proceed to the photographic rectification without necessity of re-establishing the inner orientation of the photogram with apparatuses like the transformer of Carpenter, employed basically in cartography, formerly to informatic revolution.

Its operation is based precisely in locating the three series in such a particular manner, named properly

position of Scheimpflug, that permits re-photograph nadiral plates with different origins for the first perspective (nadiral plate / rectified image, but focal distance no controlled), and the secondary (nadiral plate / rectified image and camera focus prefixed).

The figure 3 represents the series as rectilinear series of points and therefore a product between first category shapes has been considered. This correspond to the real situation tridimensional by means of the intersection of the system by a plane that contains the centers. In this case the relationship between the perspectical series consist in a rotation of the perspective center  $O$  until position  $O_1$  in base to a series of geometric conditions that exceed or purpose. This revolve, by virtue of a product of projectivities, makes useful the process.

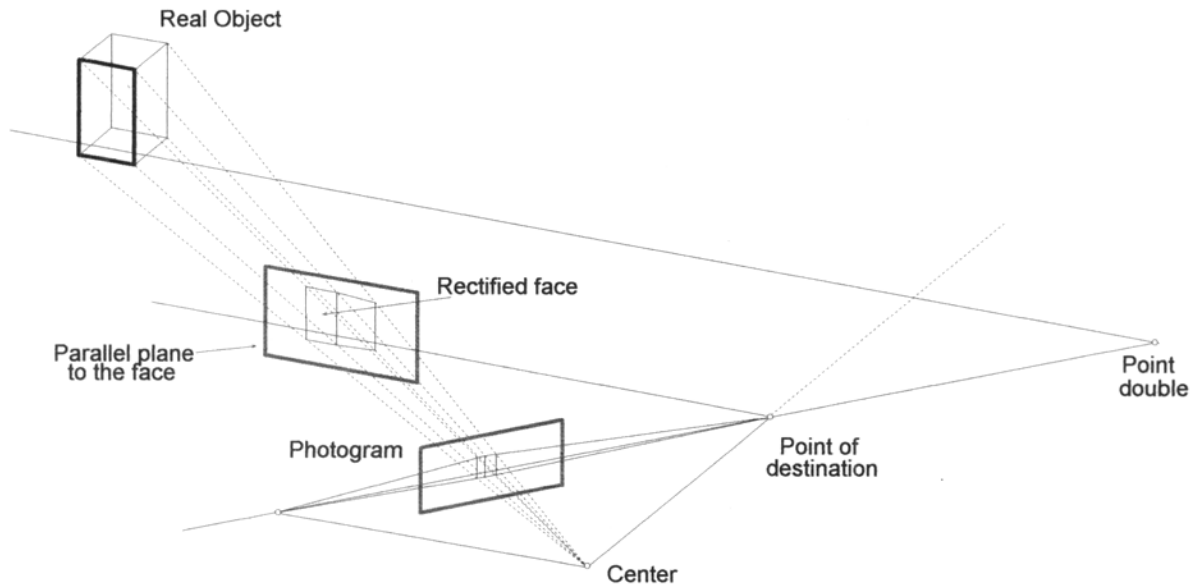


Figure 4: Fundament of the graphic rectification of photograms.

### Geometric Rectification.

From these elementary fundamentals is possible also proceed to the graphic rectification of photograms isolated by means of computer applications based in vectorial drawing, emphasizing the authentic geometric nature of the question, and implementing a working method that for its simplicity could facilitate the resolution of a good number of practical problems of representation<sup>2</sup>.

These applications are characterized by presenting the outputs of analytical calculations as drawing entities called primitives bi-dimensional or tri-dimensional, so that the precision of the graphic operations that we carry out with them can be considered equal to the corresponding numerical calculations that we could propose with its algebraic equivalent entities.

With this approach is possible the reconstruction of the space model associated to the original photographic plate of the real object, working in virtual space with the image of the own digitalized photograph, inside the module of tridimensional representation of the informatic application, and after generate an homotetic perspectivity -that means parallel intersection of the bundle- with the real object face without changing the origin point. This will be geometrically equivalent to a rectification of the initial plate. (Figure 4)

The informatic treatment of rectification also permit eliminate all a quantity of difficulties that photographic rectification system has presented traditionally derived of the method for re-photograph the initial take. For that we will depart from the spatial position of the photographic / geometric center obtained starting from data like camera focal distance and/or some geometric features of the model as perpendicularity or parallelism. In this case it is possible to generate a digital perspective of the image in what the destination point belongs to the perpendicular

direction to the of object face, so that this will result parallel to the planar face of the object to rectify,

because the informatic application will generate a conic perspective precisely in the perpendicular plane to the main direction of the object planar face determined by center and point of destination.

In the example of figure nº 5 is represented an external view of the informatic model obtained from a digitalized photographic plate transformed in a material assigned to a slab with null thick, and the same size in pixel than the digital image, and finally rendered in cylindric system. Supposed well known that the façades of the little pavilion are perpendicular its is possible drawing the geometric construction that allows to obtain the position of the center V from the points  $F_1$  and  $F_2$  related with the horizontal directions and  $F_v$  with the vertical. Achieving the necessary rotations results determined with redundancy the focal distance VP, and this permit to place the center point V in his tridimensional position. The employ of metric photographs permits the comparison with the focal distance obtained and a null incidence of radial deformation.

Once obtained the former tridimensional model it is possible to transform the original image in a photograph without vertical deformation, generating the perspective with C as destination point (VC belongs to the horizontal plane that contains V,  $F_1$  and  $F_2$ ) as shown in figure 6. In the same way if destination point was  $F_1$ ,  $F_2$  or  $F_v$  the rectification would correspond to the left, right, and lower faces of the little pavilion respectively. (figs. 7,8 and 9)

A development more detailed of all the questions that we have sketched briefly here will allow fathom in the importance of these basic concepts for to get the maximum benefit of these potent gears of work that we dispond today. Problems of a disperse nature as those that happen in the world of the technical representation, emphasize the importance of the geometric methods, like many time ago has written David Hilbert.

<sup>2</sup> Our experience is based in Bentley's Microstation.

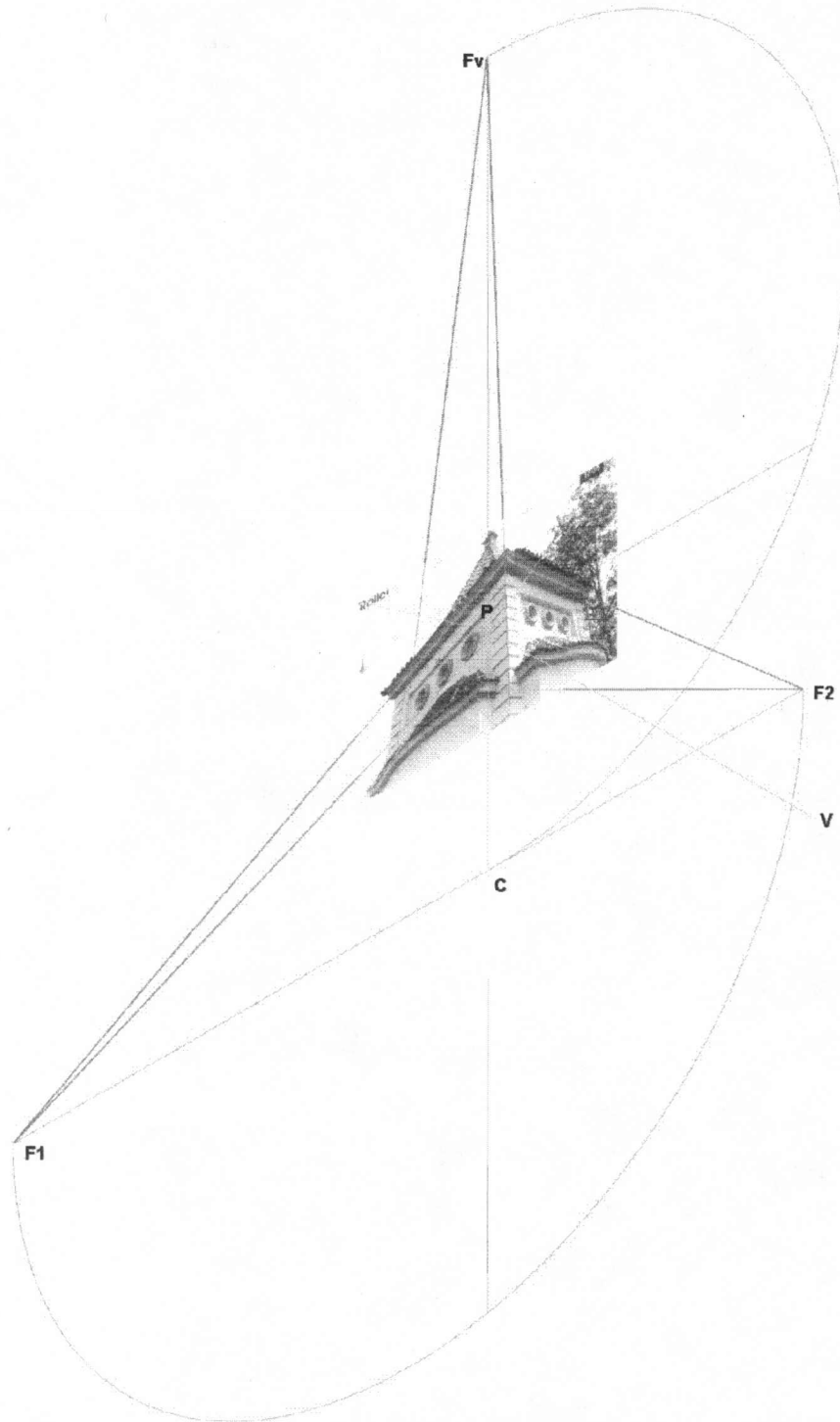


Figure 5: Informatic tridimensional model for rectificate of a photogram. The coordinates of the singular points (origin, main, and imaginary) can be obtained from geometric approaches supposed the model adjusted by tri-rectangular axis.

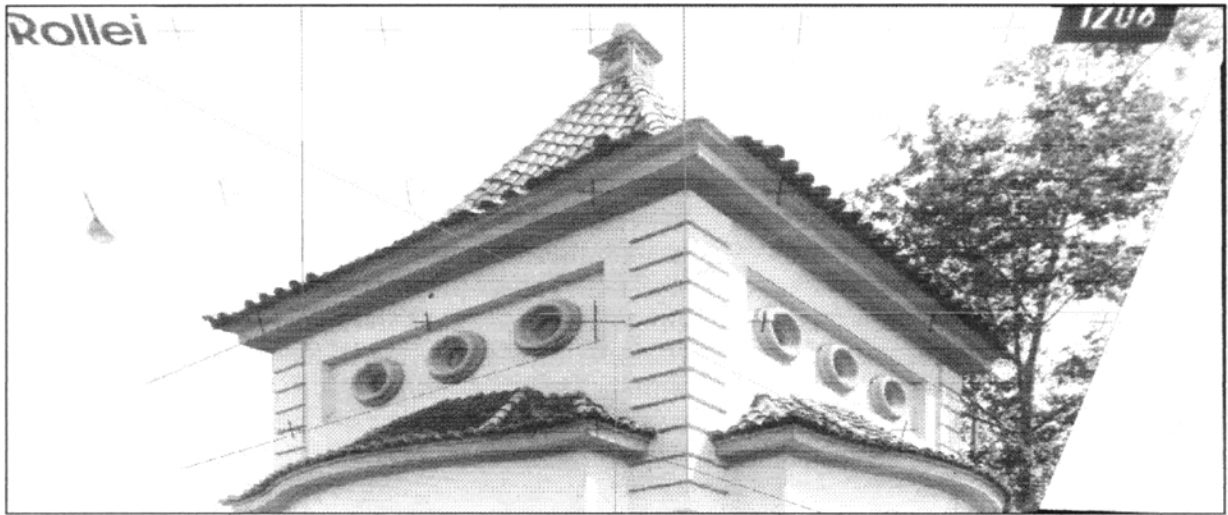


Figure 6: Rectification of former photograph with elimination of vertical conicity (main distance V-C).



Figure 7: Rectification of left side of the little pavilion (main distance V-F<sub>2</sub>).

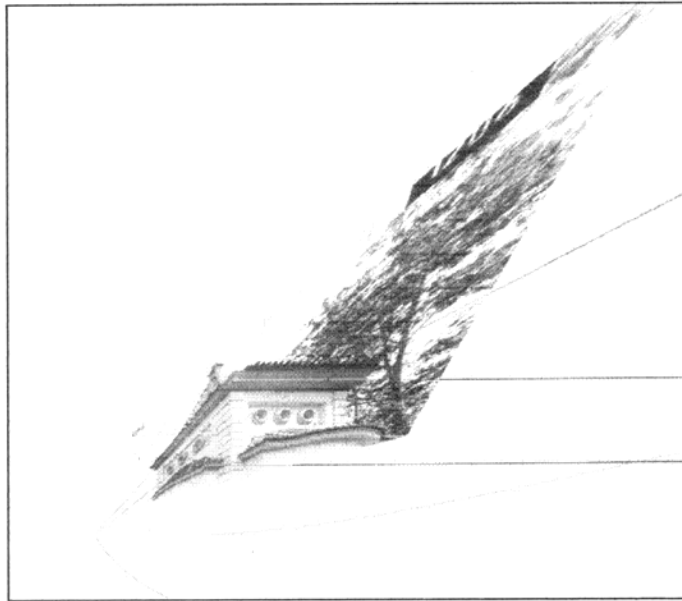


Figure 8: Rectification of right side of the little pavilion (main distance  $V-F_1$ ).

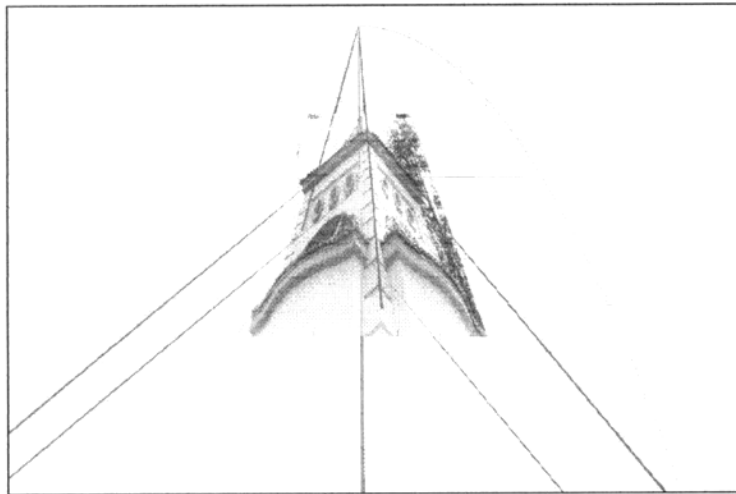


Figure 9: Rectification of the lower side of the little pavilion (main distance  $V-F_v$ ).

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