STRAIN ANALYSIS TO TEST HISTORICAL SURVEYS

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Abstract

Usually testing operations are used with the aim to evaluate the metric and semantic content of drawings of a just realized survey, concerning both cartographic and architectural scale. But hardly testing operations are tackled when we have to consider some existing surveying drawings in terms of their re-use in a conservation project. We have often to work with graphic plans, sections and elevations, that were made in the past and the goal is to understand their accuracy, and if it's possible to use them again, thereby avoiding new surveys campaign.The quality of existing surveys and their possible accuracy can be estimated by using analytical tools that the authors have already adopted for the geometric knowledge of the historical cartography: the use of plane transformations and strain analysis.

The first method is based on the analyses of conformal plane transformation results and on the comparison between distances of two sets of corresponding points, the first recorded on the monument, while the second proceeds from the papery drawings.

Regarding strain analysis, elasticity theory can provide important information about the geometric differences between two representations of the same object or between a representation and a set of measured points recognized in the drawing.

This means to search for local deformation parameters, calculated in specific areas of the drawings. Introducing invariant quantities and more analytical treatments, they may, in quantitative terms, offer a more objective description of changes in the geometric content of a survey compared to another.

Tests on preexisting surveys aim to evaluate metric and semantic contents of the graphics of readily available documentation, in order to use them for preservation projects.

Often, in the attempt to reduce the costs related to restoration and preservation works, surveys are reduced to a minimum, with consequent gaps in the cognitive process of the architectural structure under examination.

The testing procedures described below follow this perspective: intervening with an in-depth survey campaign is not always necessary; surveys can be reduced instead, in favor of the restoration of the existing documentation. This approach allows the cutting of expenses, without giving up on the metric knowledge required of the building, thanks to the evaluation of the precision of the documents available.

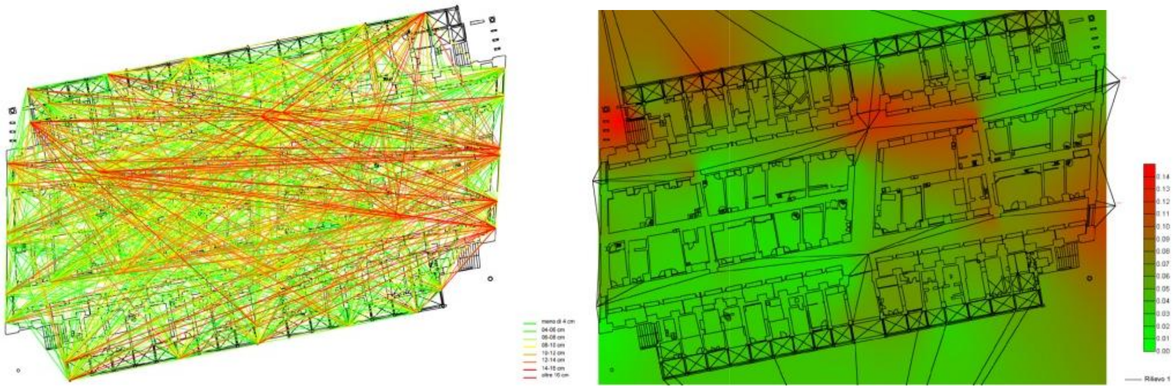
Plane transformation and residual analyses

The test operation is based on the comparison of two sets of corresponding points: one registered on the architectural structure at issue, the other one obtained from the existing drawing (plan or view).

The points on the building can be topographically mapped through polar coordinates from the vertices of the main grid in order to guarantee their precision, minus the instrumental errors.

The points distribution has to be as homogenous as possible, to prevent the concentration in a specific area from negatively affecting the general evaluation of the map.
The sets of the points obtained will be used to calculate and analyze the data, similarly to what will be described in the following charts (figure).
The suggested analyses are based on the analysis of the outcomes of a conformal transformation of the plane and on the comparison between the distance of points in the two sets. Most of the time, these analyses are sufficient to evaluate the metric property of the digitized existing surveys.

The first testing procedure consists in the comparison of the coordinates of the map with the coordinates of the terrain. The map coordinates are transferred on the survey's reference system with a similarity transformation (four-parameter transformation: rotation, X-axis and Y-axis translation, scale variation). Then, we analyzed the coordinates' differences and the module of the difference vector between every roto-translated point on the map and its corresponding point on the terrain. This procedure allows an overall evaluation of the map through the parameters of the similarity transformation, as well as the exact localization and quantification of the errors through the differences of the coordinates.



Test of papery drawings of architectural survey of palazzo della Ragione in Padua by using plane transformation

The second comparison is between distances: from each point on the map and on the survey we calculated all the distances from the other points in that group. Therefore, we obtained two sets of distances, one on the map and the other on the object, that we compared evaluating the differences between corresponding distances.
The two procedures are complementary. The first procedure allows us to localize errors, to build maps of errors and to evaluate the direction of the errors. However, it is subject to the influence of the distribution of the points on the monuments and it is also subject to masking effects of the errors due to the computation of the least squares used to assess the transformation's four parameters. The second procedure has the advantage of being untied from a particular reference system and, moreover, allows us to evaluate the relative errors between different areas of the monument. However, it does not permit an exact assessment of the error, but it only signals its presence. The usage of both procedures on the same sets offers the possibility of taking advantage of their positive characteristics, overcoming their limits. In fact, redistribution and masking phenomena of the errors are easily identifiable using the analysis of the distances. If the two procedures lead to similar results, we can reasonably assume that those results are truthful.
To solve further uncertainties about the accuracy of the surveys and to run further tests, one can employ the strain analysis described below.

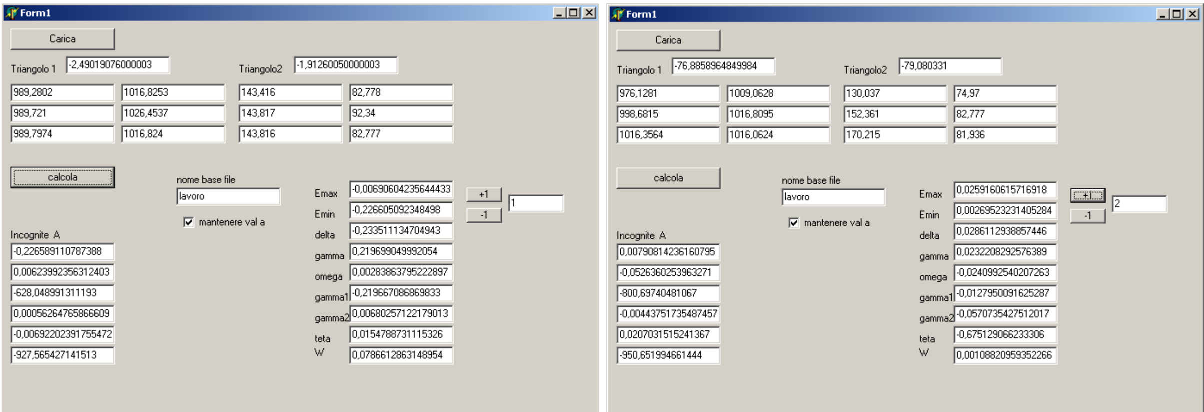
Strain analysis for the geometric comparison of maps

Elasticity theory can provide further information regarding the geometric differences between maps, compared to the analysis of the residuals obtained from the best fitting between current cartography and historical maps, which has been run to determine which reference system best matches the system hidden in historical maps. Elasticity theory implies the investigation of the local deformation parameters, calculated in specific areas of the maps.

Introducing invariant quantities and other analytic processes, we can obtain a more objective, quantitative description of the distortions of the geometric content of a map compared to the other.

The method suggested uses the differences of the coordinates of common geometric elements (features) in the maps, once the translation, rotation and scale differences between the two sets of points defining the features have been eliminated; which means after having performed the best fitting on the two sets using a roto-translation with a scale variation. In abstract terms, the coordinates' differences are considered as if they were the differences of coordinates of points on an "body" in two different stages: an unstrained status, corresponding in our case to the reference map; and one under strain that in our case corresponds to the historical map, whose projective content is unknown.

Once established this “mechanical” correspondence, the strain analysis describes the behavior of a map relative to the other, hence showing the geometric differences.



Screencapture of the software to calculate elastic parameters

Strain_Map software

To perform the strain analysis, we implemented the *Strain_Map* software, which allows the calculation of calculate the elastic parameters starting from two triangulations based on two sets of homologous points on two separated maps.

Through the interpolation and the mapping of the corresponding values, it is possible to assess the differences between cartographies. In fact, the values of the resulting parameters are assigned to the barycenter of each triangle; for all the other points, the values are obtained through interpolation using Kriging algorithm.

The software calculates the system variables, by uploading the file with the data of the triangulations on the two maps:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix} = \begin{bmatrix} x_{11} & y_{11} & 1 & 0 & 0 & 0 \\ x_{12} & y_{12} & 1 & 0 & 0 & 0 \\ x_{13} & y_{13} & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_{11} & y_{11} & 1 \\ 0 & 0 & 0 & x_{12} & y_{12} & 1 \\ 0 & 0 & 0 & x_{13} & y_{13} & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \alpha \\ \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \\ \beta \end{bmatrix}$$

that is, the partial derivatives $\partial u/\partial x$, $\partial u/\partial y$, $\partial v/\partial x$ e $\partial v/\partial y$ and the values of α and β. Once that the values of the variables have been found, we can determine the following strain parameters:

1. E_{max}
2. E_{min}
3. expansion $\Delta = E_{max} + E_{min}$
4. maximum strain $\gamma = E_{max} - E_{min}$
5. rotational distortion ω $\omega = \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$
6. shearing stress $\gamma_1 = e_{xx} - e_{yy}$ *e* $\gamma_2 = e_{xy} - e_{yx}$
7. azimuth of the axis of maximum strain $\vartheta = \arctan(\gamma_2/\gamma_1)/2$
8. strain energy $W = \Delta^2 + 1/2\gamma^2$.

The output files of Strain_Map software are:

- a table with the barycenter coordinates and for each triangle the values of variables and parameters of each triangle;
- a .dxf file of the triangulations run on the two maps.

Delaunay Triangulation

Starting from the control points (topographically surveyed), the plan area is automatically divided in triangular patches, using specific triangulation algorithms. Among these, *Strain_Map* uses Delaunay triangulations. The triangulation identified on the historical map is transferred on to the current map using homologous points.

This means to properly connect the points of a set in order to create a set of triangles, minimizing the sides' lengths.

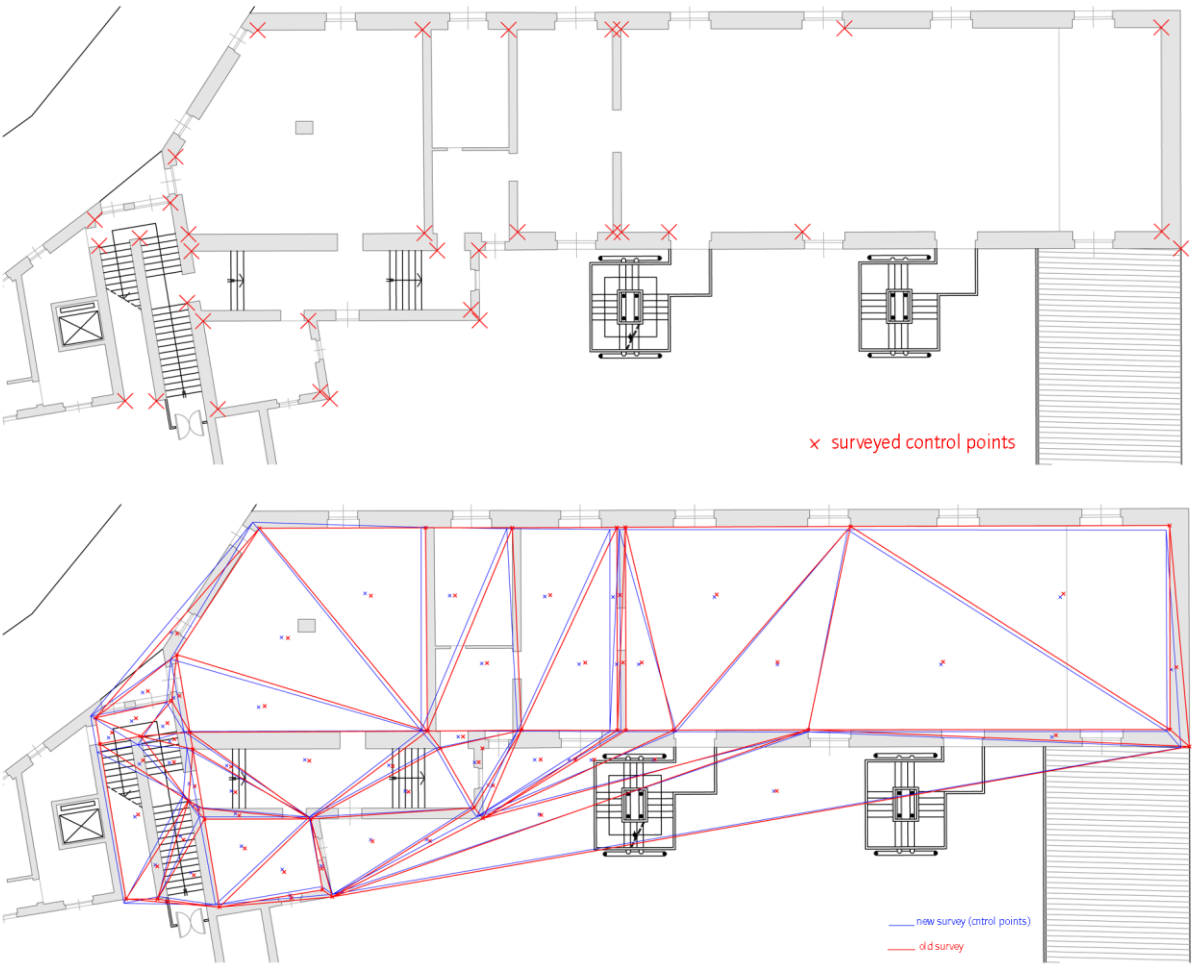
Given a set of bidimensional points {P}, a Delaunay triangulation consists of a set of triangles having a set {P} of points as vertices, such that for each one of them, no point in {P}, aside for the vertices, is inside the circumcircle of any triangle.

If there are only three points in a circle (the vertices of each single triangle), the Delaunay triangulation is unique. If there are more than three points on the circle, we can still obtain an univocal triangulation.

The algorithm computes the triangulation from an initial, arbitrary triangle that includes all the points.

Each point is introduced by observing which triangle in the set has a circumcircle containing the new point. This is determined by comparing the radius of the circumcircle with the distance between the new point and the circle center.

Those triangles whose circumcircles intersect are delated and replaced with new triangles that include the new point. New triangles are hence created starting from the new point and the remaining sides.
In this way, for each newly introduced point the number of triangles increases by two. When all the points have been introduced, we obtain the final configuration of the triangulation. All the triangles are *Delaunay triangles*: that is, their circumcircle does not contain any point.
The application has been computed on a current survey, adopted as a reference map, and an existing survey of the building complex of the Tolentini, historical venue of our University. This application provides more information than the interpretation of the deformation analysis based on the residual distribution, because it further highlights the geometric local anomalies and compares it with the whole geographic representation.

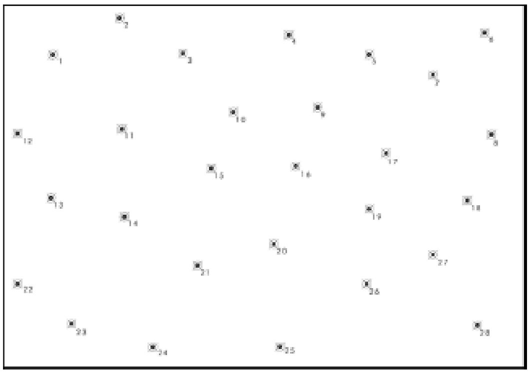


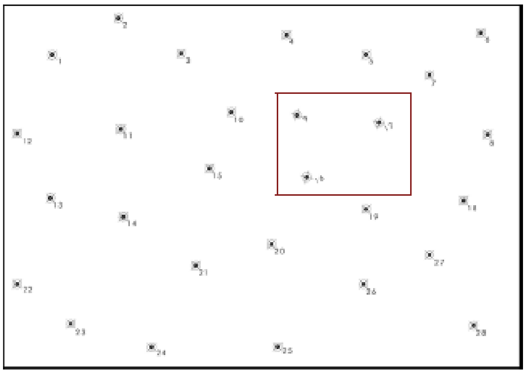
Test on the Tolentini old survey: Delunay's triangulations on the 2 compared surveys

Therefore, it highlights possible rotations of some parts of the existing survey (common error of a direct method survey) compared to the current survey.
Once we compute the triangulation from the control points and we assign to the barycenter of each triangle the calculated, corresponding strain parameters, we can draw isolines of same strain value (iso-Δ lines for the expansion, iso-γ lines for the maximum shearing stress, iso-ω lines for the rotation and iso-W lines for the dissipated energy), which in a sense show the variation due to the different geometry of the maps.

The tests

Before proceeding with the computations on the survey plans, we ran a test to verify the software reliability.

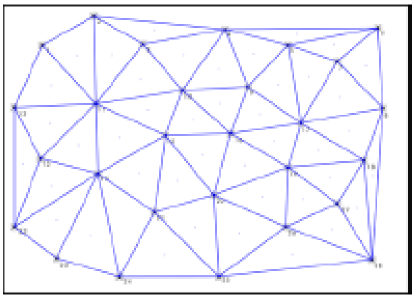


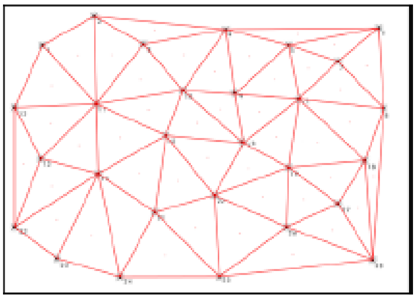


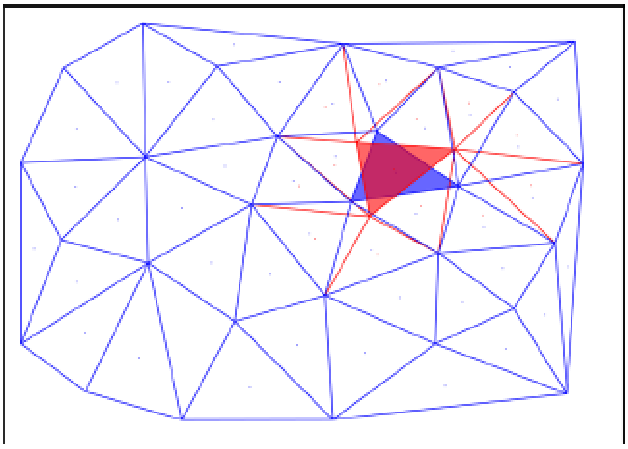
As shown in the previous figures, we generated an initial image with 28 signalized points homogeneously distributed.

Then, we generated a second image similar to the first, but with three out of 28 points rotated counterclockwise by p/6, that is by 0,523

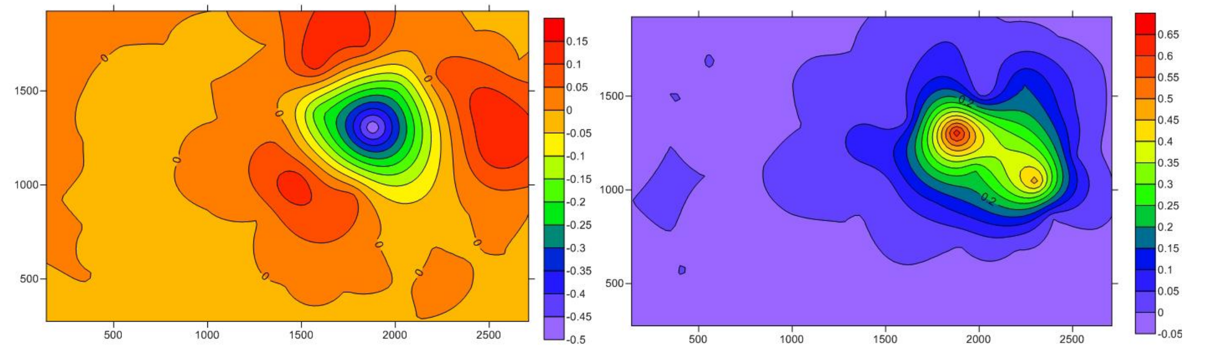
The two sets of coordinates of the points in the previous images have been used as an input for the software, which calculated the parameters of the strain analysis and generated a file with the triangulations on the points sets.





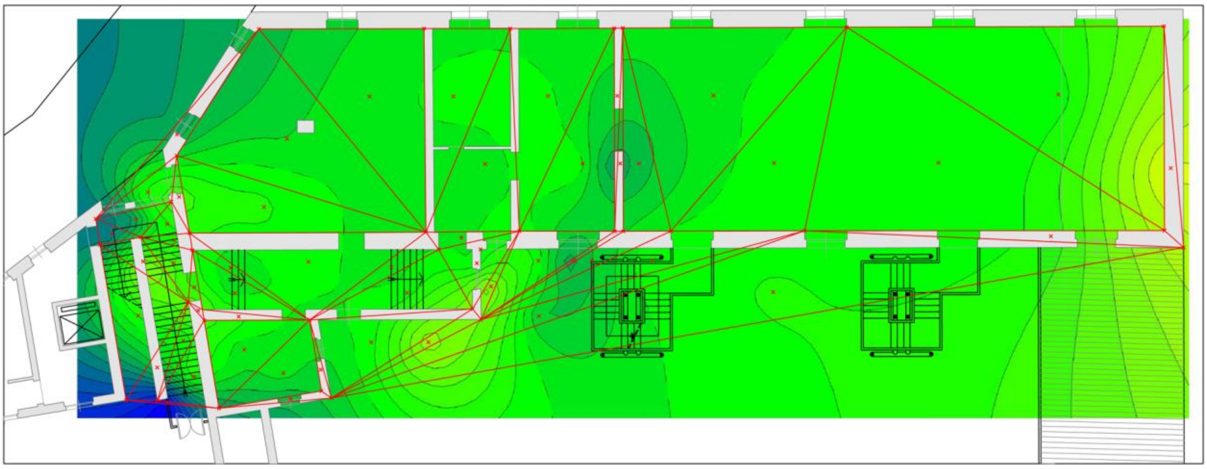


Overlapping the two triangulations, the rotation of the marked triangle can be easily seen

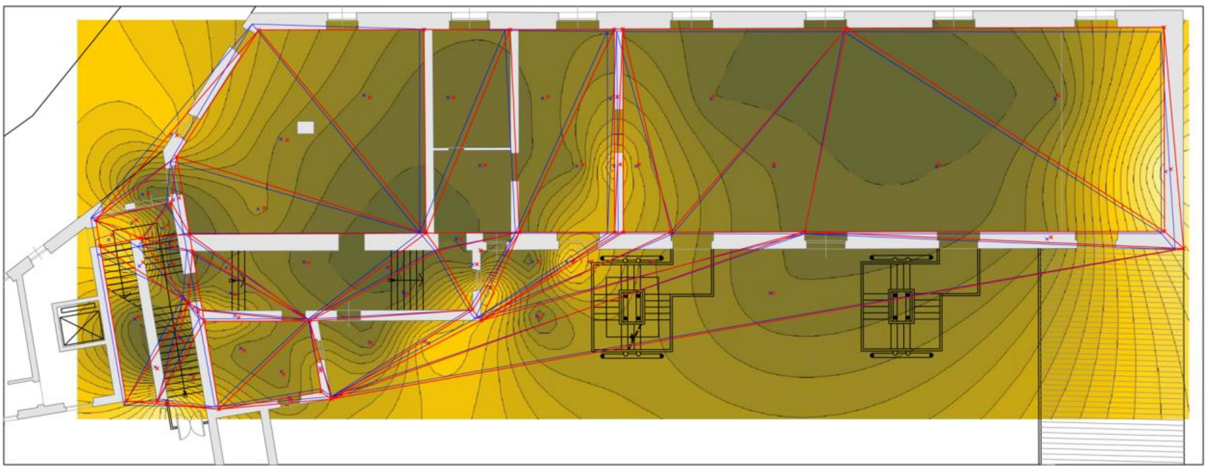


Left image: mapping of the rotation parameter ω. The point of maximum rotation corresponds to the barycenter of the rotated triangle. Parameters calculated for that point is -0.5, which corresponds to π/6, that is the rotation initially applyed.

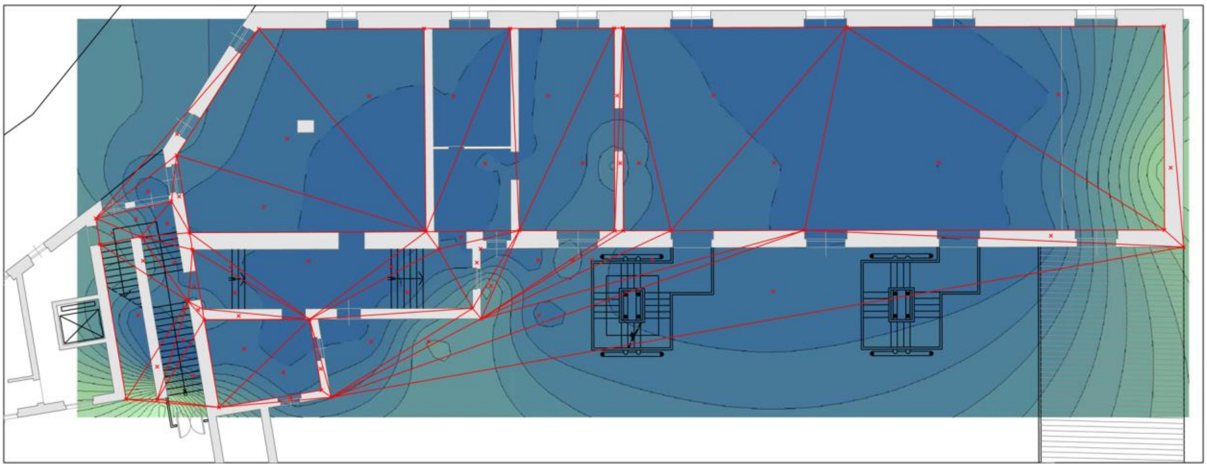
Right image: mapping of the parameter W, the energy of maximum strain.



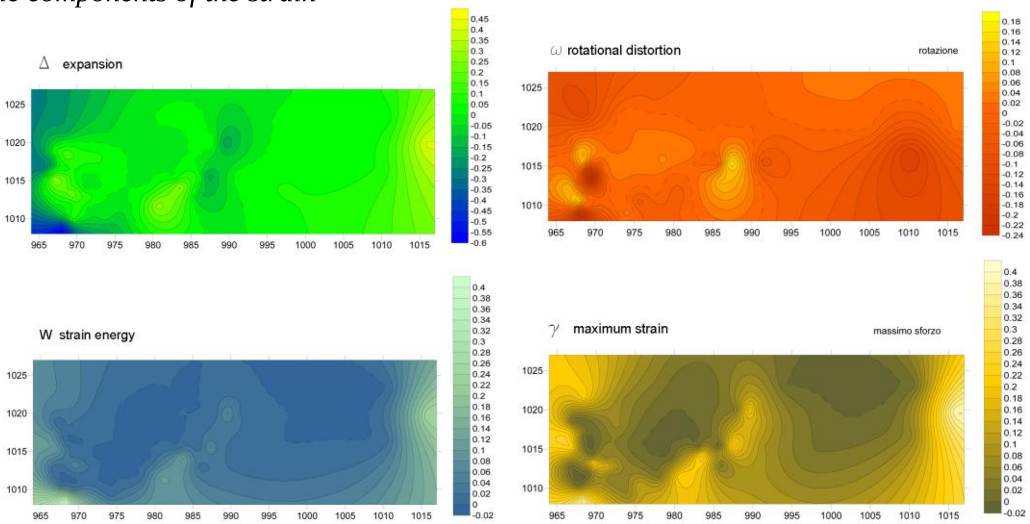
Tolentini control test: isolines representation of the isotropic deformation due to the expansion values (error in scale digitation)



Tolentini control test: isolines representation of the anisotropic deformation due to the maximum shear values



Tolentini control test: isolines representation of the W energy which summarize the anisotropic and isotropic components of the strain



Summary of the main parameters

Conclusions

With this experience we want to demonstrate that, through the evaluation of the outcomes of the analyses performed, it is possible not only to verify existing surveys, but also to plan a procedure for their geometric preservation/restoration.

The authors performed an in-depth testing in cartography and, therefore, it can be applied on historical surveys as well as on surveys performed with non-rigorous methods.

Once identified possible localisms in the preferential distribution of the error, the drawings can be "corrected" using a procedure of local deformation based on the use of warping algorithms. These algorithms are well known in computer graphics and have been already used in the preservation of historical maps.

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