

PHOTOGRAMMETRIC METHODS APPLIED TO THE REPRESENTATION OF CURVE FACADES: AN EXPERIMENT CONDUCTED ONTO PALAZZO CARIGNANO IN TURIN

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KEY WORDS: Photogrammetry, Representation, Architectural, Geometric Transformations

ABSTRACT

The dealt topic is the representation of a curve building façade; the experiment has been conducted onto the Palazzo Carignano façade in Turin. Façades with a curvilinear plant and a prospect characterized from projections out of the surface cannot be developed on a plan simply developing a cylindrical surface on control points (GPC) and second or third order polynomial rectification. In order to correct perspective deformations of the three-dimensional projections, the classic ortho-photogrammetric method has been used, inserting a polar to Cartesian coordinate transformation into the calculation procedure, following therefore the canonical passages of inner and outer orientation, with the use of a 3d digital model realized through data-acquisition from laser-scanner. The used procedure concurs therefore the rigorous verification of the accuracy through the appraisal of the residuals, otherwise not possible using control points to guide a simple deformation, because a little deviation of the control point from the reference surface can cause a wrong deformation of the whole image. The program-module of rectification has been modified, introducing the transformation of polar coordinates (x-axis as curvilinear development, y-axis parallel to the cylinder axis and z-axis in radial sense) in Cartesian coordinates; using the Cartesian coordinates it will be therefore possible to trace the photographic image of an ideal cylindrical development plan. The final photographic mosaic therefore will be characterized from x-axis as development of a circumference arc that concurs to bring back in scale only the ideal surface of the object, the nearer or more distant parts from the axis of the cylinder finding themselves on arc of circumference whose development is smaller or greater than the reference surface. The measure on axis x in fact represents the angle of the polar reference system. The advantages of this type of method consist in obtaining the development of a surface curve also of objects with a 3d development (as it is the usual case in architecture), using data that would be acquired for a traditional architectonic ortho-photogrammetry, but whose final product contributes to represent a curve surface in the most complete way. Moreover the acquired control points can prescind from effective geometry of the object, since the transformation of the projection from orthographic to cylindrical is calculated on the base of outer orientation, an ideal geometry of one or more reference cylinder and a real 3d elevation model.

1. INTRODUCTION

The survey of Carignano Palace's seventeenth-century façades, in Turin, has been carried out to have a metrical reliable and detailed representation; another main request was to have a kind of representation suitable to support a campaign of descriptive and decay analysis. Photogrammetry and imagery processing can be considered appropriate to answer these requests.



Figure 1. General view of the facade.

Since photo-rectification and ortho-photo projection cannot be applied to a curved surface for different reason (rectification cannot be applied to curved objects, ortho-photo may be affected by foreshortening if the represented object isn't parallel to the projection plane), it was decided to search a method to develop the curved object using a reference mean curved surface.

A curved surface perfectly fitting a mathematic surface (cylinder) can be developed using polynomial transformation starting from a planar representation derived for example from ortho-projection. Such a procedure will not consider recesses or mouldings, or even physical imperfection of the object if compared to its mathematic representation.

If polynomial image transformations can be suitable for fresco or mosaic surfaces, architectural façades characterized by pilasters, columns, capitals, mouldings cannot be considered somewhat planar. It was then necessary to apply orthographic projection to a curved surface: infinitesimal areas had to be projected over a curved surface, and then the curved surface could be transformed by developing it into a planar surface.

2. DATA ACQUISITION

2.1 Instruments

It has been necessary to acquire three kind of data:

- photographs;
- topographic control points;
- 3d model of the façades through laser-scanner.

Photographs had been acquired using a digital camera (canon EOS20D) equipped with two fixed-focal lens, 50mm and 85mm, which guarantee very low radial distortion (according to MTF tests published and verified through auto-calibration) and optimal resolution.

The complete coverage of the façades required the acquisition and processing of approximately 800 images. Optical axis of the camera was always perpendicular to the façade: it has been used a 25m aerial-platform to reach every necessary level.

Topographic control points had been acquired using a reflector less total station, spread all over the surface and used to

calculate outer orientation of photographs through space-resection. Finally, the three-dimensional model of the façade has been measured using a laser scanner HDS 2500 (Leica geosystems) in order to obtain a full representation of reliefs and curves. The point-cloud was acquired at a mean resolution of about 1 cm; after model registration and orientation, the cloud was sampled down to a 2 cm mean resolution, more than adequate to realize the digital elevation model required.

3. DATA PROCESSING

3.1 Mathematic transformation between geometrical spaces

The central part of the main façade of the palace is built on a geometric scheme made by several ellipses joint together through tangent points. Due to the analytical expression of ellipse, we had to find a simpler geometry suitable for the calculation process. Ellipse is defined as the locus of all points in the plane the sum of whose distances r_1 and r_2 from two fixed points F1 and F2 (the foci) is a given positive constant $2a$:

$$r_1 + r_2 = 2 \cdot a \tag{1}$$

Given that the length calculation of an elliptical arc is a hard mathematic task, although subject to approximation, we decided to approximate *a priori* ellipses to arc of circles, in order to use simple trigonometric functions rather than integrals.

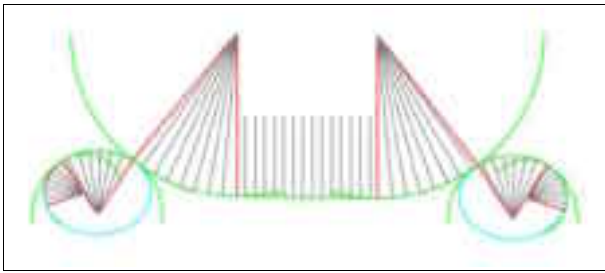


Figure 2. Central part of the façade subdivided into six cylindrical projections and one central ortho-projection

Figure 2 shows a simple geometric scheme of the various portions of circular arcs developed and the original ellipses found as building's fundamental geometry. The difference between ellipses and circles on the right part is shown in the following table:

arc num. 1		arc num. 2	
degrees	difference	degrees	difference
0	± 0,0000 m	0	± 0,0000 m
10	+ 0,0085 m	10	+ 0,0117 m
20	+ 0,0017 m	20	+ 0,0073 m
30	- 0,0103 m	30	+ 0,0007 m
40	- 0,0170 m	40	- 0,0026 m
50	± 0,0000 m	50	- 0,0053 m
		60	- 0,0183 m

Table 1. Difference between chosen circles and ellipse.

Using simple arcs, therefore, it is possible to use canonical equations used for developing a cylinder. Transformation equations from three-dimensional space to cylindrical space, with reference to a cylinder which is tangent to the building surface are as follow:

$$\begin{aligned} x_c &= r \cdot \arctan \frac{x}{(r-z)} \\ y_c &= y \\ z_c &= \sqrt{x^2 + (r-z)^2} - r \end{aligned} \tag{2}$$

or as:

$$\begin{aligned} x_c &= d \cdot \arctan \frac{x}{(r-z)} \\ y_c &= y \\ z_c &= \sqrt{x^2 + (r-z)^2} - r \end{aligned} \tag{3}$$

where:

- x, y, z are the coordinates in Cartesian unmodified space;
- x_c, y_c, z_c are the coordinates in cylindrical space;
- r is the radius of the reference cylinder;
- d is the distance from centre to point.

Axis are laid in this way: in Cartesian space, x axis is parallel to the façade, y axis is vertical and z axis is outgoing from the façade. In cylindrical space we use a polar coordinate system with x axis as curvilinear development, y axis parallel to the cylinder axis, z axis in radial direction; origin of the reference system is placed on tangent axis between cylinder and wall. The difference between (2) and (3) is in the way reliefs are treated: in the first formula (2) every point laying in a surface different from the reference one is moved along x_c axis, so to keep it aligned along the vertical axis with the neighbouring points (in other words, the final image will not be in the right scale of representation except for the reference cylindrical surface). On the contrary using formula number (3) everything will be represented in the right scale, but if the object isn't perfectly fitting the ideal cylinder, the final image will show noisy deformation under certain circumstances: for example entablatures and mouldings will be represented in the right scale, but will seem shorter or larger if seen in the global contest. This happens because an entablature in a curve facade will have a different arc length in respect to the the arc length of the neighbouring wall (shorter if concave and larger if convex). As the main requirement was to have the most useful representation of the facade for decay mapping, we chosed to use formulas (2): in this way it is possible to have a seamless transition between arcs with different radius.

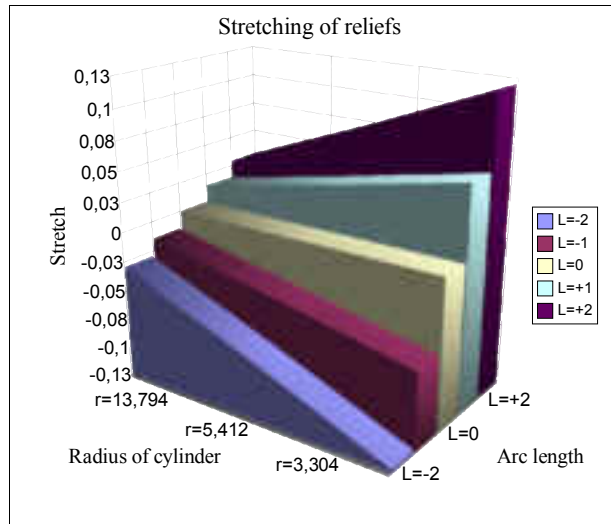


Figure 3. Stretching of elements in relief, function of radius of the cylinder and arc length.

As seen in figure 2, the central part of the façade is now subdivided into seven parts: three different cylinders on the left side, one planar central part, and other three cylinders on the right side. Each curve was evaluated to best fit the real geometry, in reference to main wall “plan”. The central part, characterized by a vault and concave and convex parts, was too complex to be developed in separated parts, so it was decided to execute a simple orthographic projection.

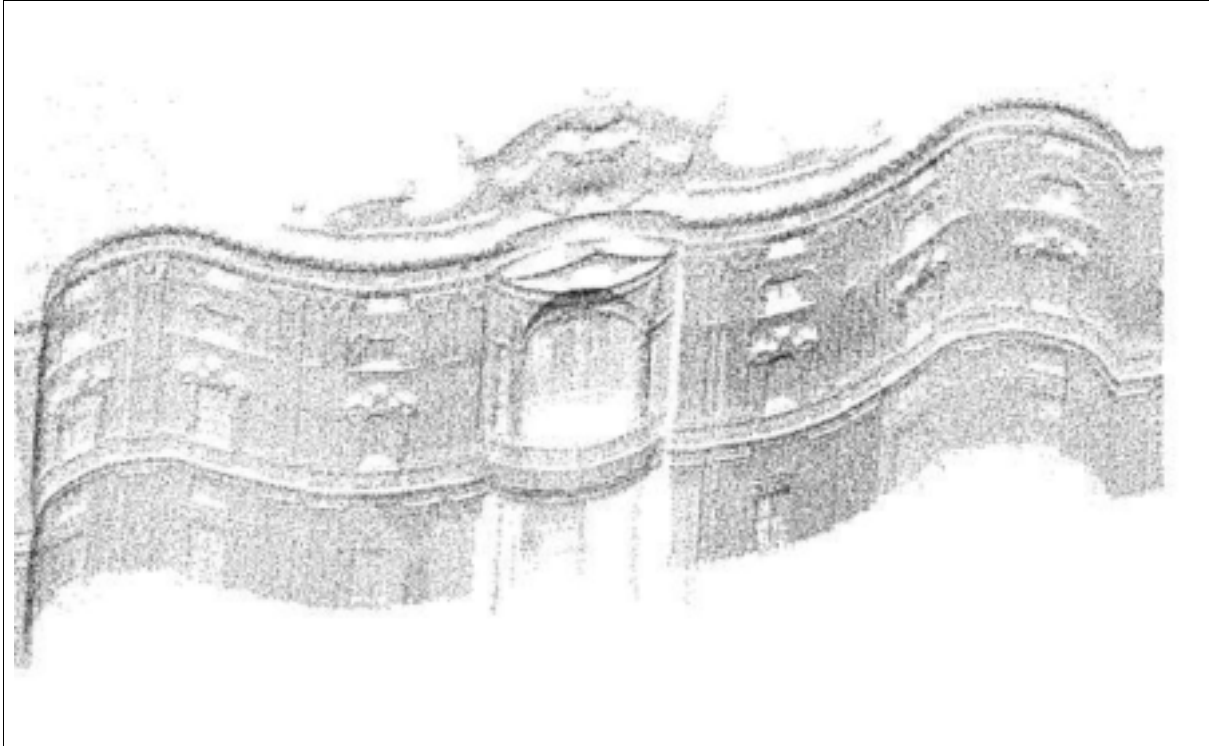


Figure 4. Simplified part of the original point cloud.

Each of the parts described above was separately processed and only in the end final results have been composed in a seamless mosaic.

3.2 3d data manipulation

DEM data was to be modified in order to obtain a three dimensional map of reliefs. Equations (2) have been used to transform the original point cloud in such a way to represent a cylindrical developed space. One translation and (eventually) one rotation have been applied to point cloud to bring the tangent axis into the origin, thus making possible to use the simplified equations in (2).

Six developed point clouds and part of the original point cloud was to be imported into GRASS GIS software, used to manage all acquired data. DEM surfaces, considering density of the cloud, was generated by interpolation using a simple inverse distance weighted algorithm.

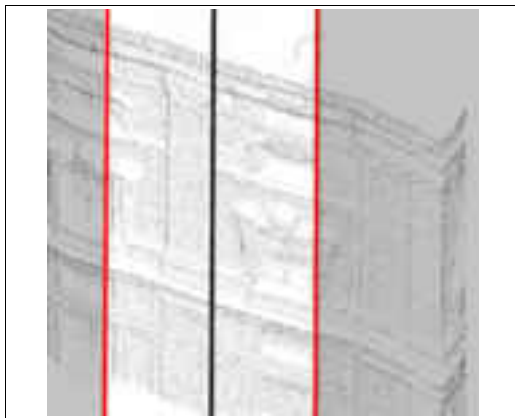


Figure 5. Particular of a developed point cloud.

First of all, a translation was to be imposed to the cloud in order to bring the origin of the reference system along an ideal axis on the external surface of the reference cylinder. This was

necessary to significantly reduce computational overload during photo-projection.

The 3d point-model was then developed using formulas (2) and single parts relative to each curvature were created (an example can be seen in figure 5). This represent the ideal image that we are going to create with final photo-projection. Using the original 3d model would have made difficult to deal with relief's shadows because of foreshortening due to curvature of the façade. Developing the 3d model and executing the photo-projection on to this DEM guarantee the best fitting between final representation and raw data.

In figure 6 is shown a DEM-image representing a cylindrical projection of the façade.

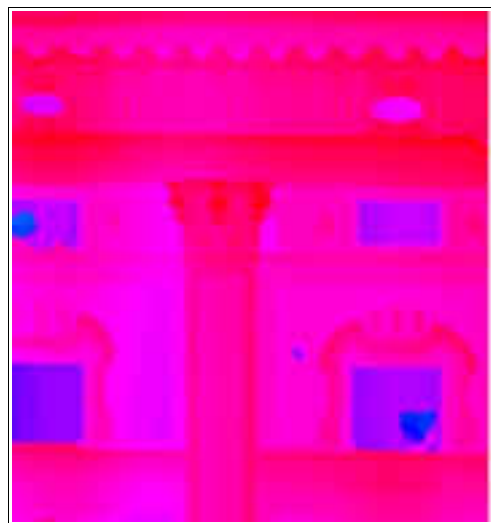


Figure 6. DEM image.

3.3 Software

Some changes had to be introduced into the software originally developed for ortho-photo rectification. We used GRASS gis

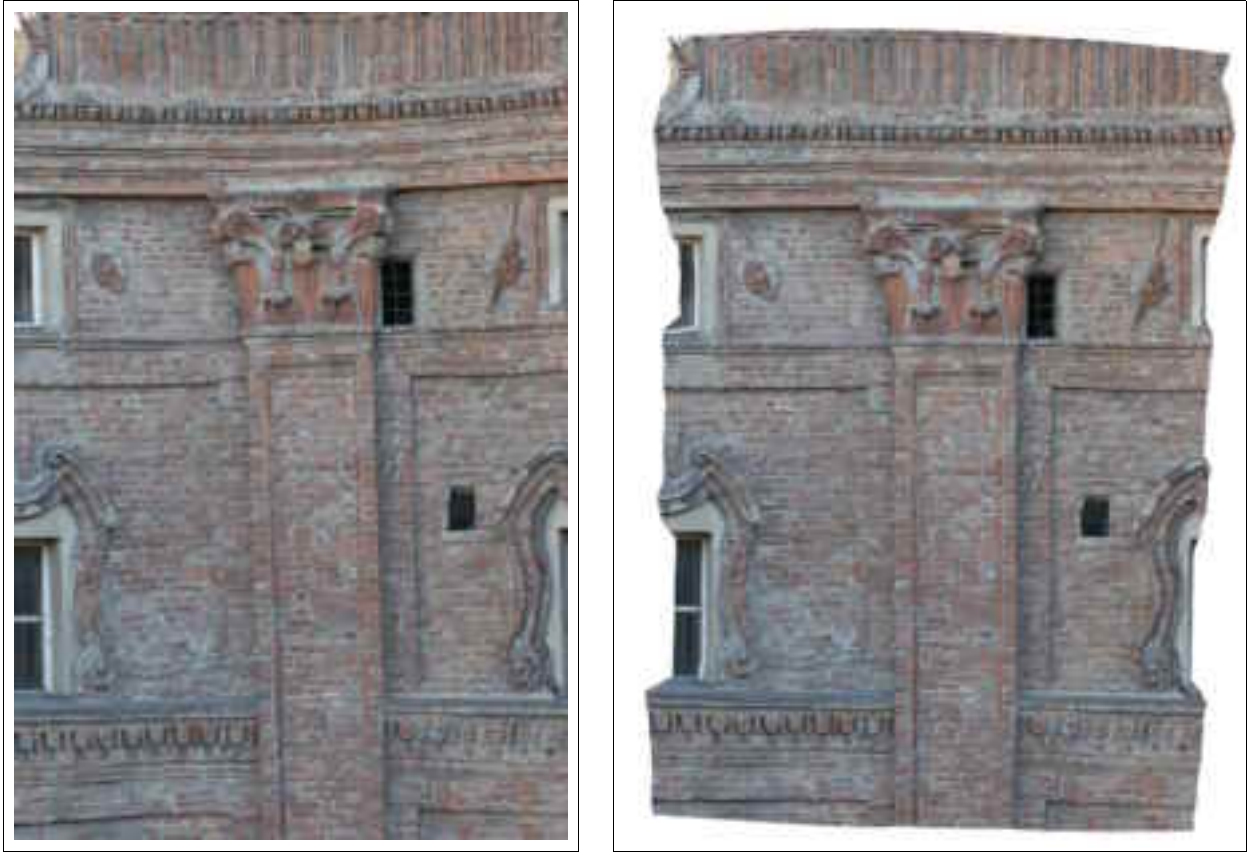


Figure 7. Original image (left) and projected image (right).

software to manage image transformation and rectification, in particular modules in the “imagery” catalogue. While there is absolutely no difference in inner and outer orientation of imagery block, during the process of rectification we had to introduce some coordinate transformation code.

The rectification module divides the area reproduced by actual image into smaller infinitesimal areas, and for each of these areas will perform a simple photo rectification. Square areas, whose dimension can be set to a value suitable to make them fit the underlying three-dimensional model, are identified by their four corners.

Rectification equations are then calculated using space-coordinates of the four corners and their corresponding image-coordinates on the orientated photograph. Since photos visualize a “curve” object, and since the 3d model has been transformed to a “planar” object, it is now necessary to transform the coordinates of the four corners from the 3d cylindrical-space into 3d original space, thus the program can recognize on the photograph the area of the curve façade that have to be transposed and represent the now considered square area.

In other words, there isn't any intermediate transformation or projection; the image is immediately transformed into a developed rectified image of the area represented.

Figure 7 shows the same image, before and after the process of projection. In the right image it is possible to notice how surface curvature has been corrected.

$$\begin{aligned} x_i &= (r - z_c) \cdot \sin(x_c / r) \\ y_i &= y_c \\ z_i &= r - [(r - z_c) \cdot \cos(x_c / r)] \end{aligned} \quad (4)$$

where:

x_c, y_c, z_c are coordinates in cylindrical space;

x_i, y_i, z_i are coordinates in Cartesian space;
 r is the radius of the reference cylinder.

Because the two spaces considered are bound by a biunivocal relation, we used (4) to calculate original space position of a point expressed in cylindrical modified coordinates.

3.4 Photo-projection and mosaic

Each image has been orientated using topographic points. Overall RMS residuals were always below 2 mm. Since the required representation scale factor was 1:20, it was necessary to operate in an adequate resolution for final printing. Image resolution of 100 points per centimetre (id est 254 dpi) was suitable for a high-resolution photo-quality printing: each image pixel represents a 2mm x 2mm area of the real object.

The main façade is approximately 70 m wide with a height of 25,5 m: a unique image of the entire façade would have been 35.000 x 12.750 pixels, for an approximate file dimension of over 1,2Gb (considering a 24bit image-depth). Since the final results was to be printed onto UNI-A1 paper sheets, it was decided to realize a single mosaic for each sheet, reducing final file size to about 100Mb each. The entire façade has been subdivided into three rows (four in the central part) and a number of columns named from A to M, for a total of 36 sheets. In the main GRASS database, defined by unique reference system, were created all the sub-regions in order to export single rectified and developed images.

Mosaics were created by manually photo-editing the composition of rectified images: due to curvature, mouldings and shadows, it was necessary to manually recognize which image best reproduces a single particular.

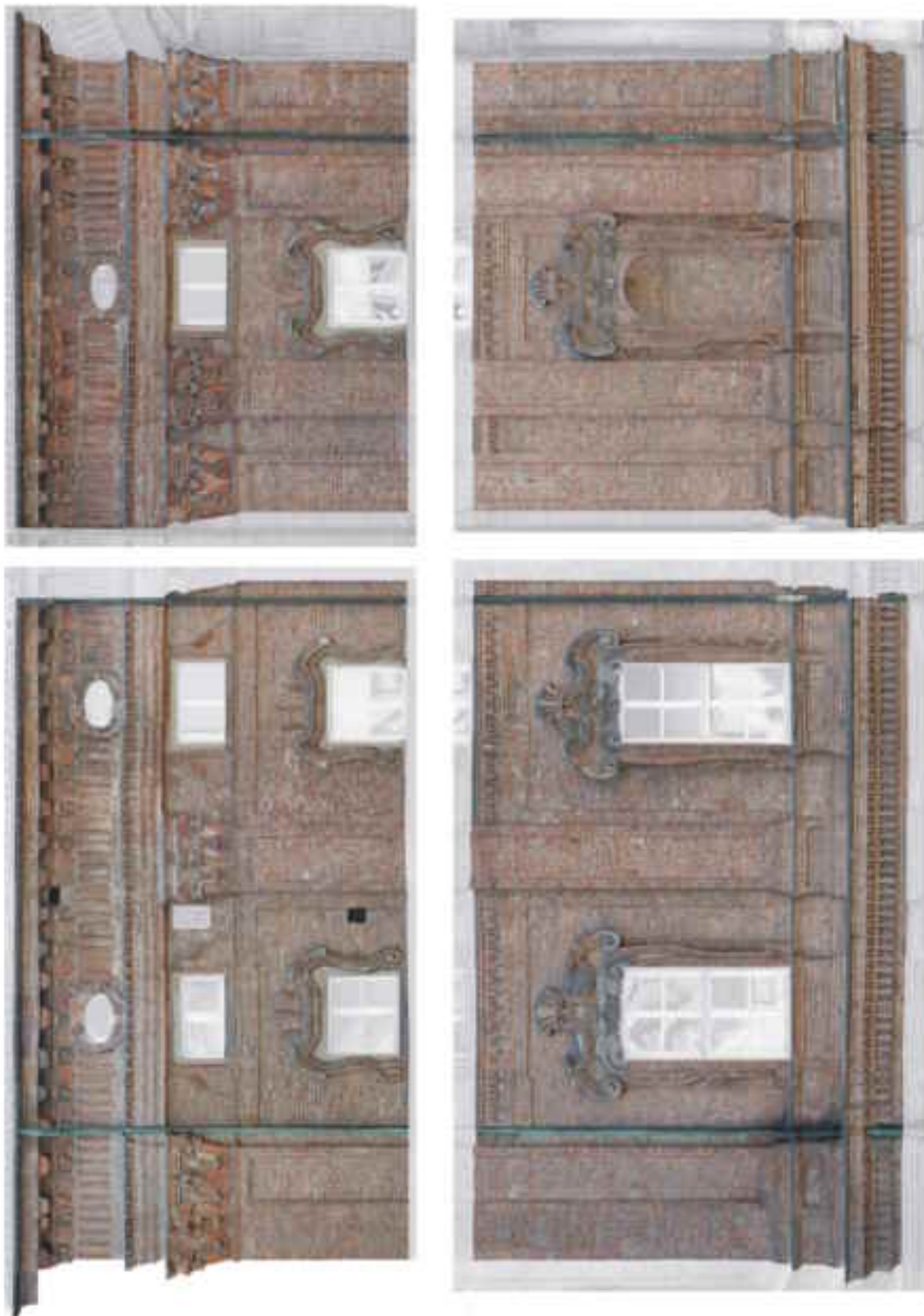


Figure 8. Example of four final mosaics of the curvilinear right part of the façade. Each sheet represents an homogeneous part delimited by architectonic mouldings. Gray (lighter) areas on edges are used to contextualize the single image.

4. CONCLUSIONS

4.1 Purposes and outcomes

This paper explains how the photogrammetric survey of Carignano Palace's seventeenth-century façades was conducted. The method of developing graphic representations of cylindrical surfaces described in (Bezoari, Monti, Selvini, 1992) has been transposed to photographic ortho-rectification, with particular care to how moulded surfaces could be represented in cylindrical development.



Figure 9. Example of nearly full-scale printout of photomosaics.

The main purpose was to have a representation useful for decay mapping (executed by restorers themselves). The mapping work was made simple thanks to the photographic printout, characterized by accurate detail description. Scale of representation chosen and high-definition printing made it possible to distinguish even different kind of mortar. At the same time, it is a metrical representation. The development of the curvilinear surfaces, at the cost of some details represented

out of scale if not laying on the reference surface, made it possible to map a complex surface as it is a planar wall, without any kind of foreshortening.

4.2 Brief comparison with other methods

The method used to realize the developed representations is derived from ortho-photo procedure, by realizing some coordinate transformations during calculation. In this way it is possible to use photogrammetric procedures of inner and outer orientation: in case of architectonic buildings this is a critical necessity, since surfaces can never be considered as perfectly fitting geometric surfaces.

Using polynomial image-warping procedures (as explained for example in D'Amelio, Emmolo, Lo Brutto, 2004) is admissible and profitable for little scale objects whose geometry can be easily defined: a building moulded façade would require the definition of a great number of warping areas each related with a single "plane". Disadvantages are for example the number of control points required since it is necessary to analyse separately each homogeneous part of the façade; second, since polynomial transformations are not bound by projective geometry like photogrammetric methods, this method would be more error-prone and less verifiable.

Using rigorous photogrammetric procedures, like space-resection or bundle adjustment for block orientation and evaluation of parameters, and simple geometric coordinate transformation during projection step, the final result can be defined as metric result (with the limitations explained) and structure deformations and deviation from the ideal geometry can be neglected as irrelevant.

Traditional ortho-photo or simple photo-rectification for smaller portions could not represent the entire façade because of foreshortening: in this case projection would have been more rigorous, with every detail in the correct scale but only in reference to a projection plane. Since these façades were characterized by concave and convex succeeding of curves, the appraisal of the surface would have been inevitably incomplete.

REFERENCES

S. D'Amelio, D. Emmolo, M. Lo Brutto, B. Villa, 2004, Il rilievo e la rappresentazione delle superfici curve della Cappella Palatina di Palermo. In: "8a Conferenza Nazionale ASITA - GEOMATICA Standardizzazione, interoperabilità e nuove tecnologie", Roma, 14-17 dicembre 2004. <http://www.asita.it/8/Pdf/0346.pdf>

Bezoari G., Monti C., Selvini A., 1992, La fotogrammetria per l'architettura, Liguori editore, Napoli.

ACKNOWLEDGEMENTS

This survey was made under request of the "Soprintendenza per il Patrimonio Storico, Artistico ed Etnoantropologico del Piemonte", superintended by Dott.ssa Carla Enrica Spantigati, which is entrusted for the maintenance of Palazzo Carignano, for which supervision is committed to Dott.ssa Alessandra Guerrini.

Entrusted architect for the survey is arch. Noemi Gallo.

We would also like to acknowledge Dott. Arch. Valentina Bonora, DINSE Faculty of Architecture, Politecnico di Torino, who helped us in acquisition, processing and use of the laser-scanner data.