PHOTO-PAIR DESIGN OPTIMIZATION
IN ARCHITECTURAL PHOTOGRAMMETRIC SURVEYS

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ABSTRACT

Photogrammetry is well recognized as the main tool in capturing and recording sites and monuments of architectural interest. Recent trends in photogrammetric research are targeted towards the optimization of the design of the photogrammetric network. Criteria used include not only the photo-pair overlap, but also the dedired optimum accuracy of the final product.

The object of this research is the optimization of a two-photo configuration for the recording of the near flat facade of an architectural monument. It is finally shown that the geometric characteristics of the optimum photo-pair configuration can be computed from relative simple "optimization polynomials", the coefficients of which depend only on the camera charavteristics and the object dimensions, thus by- passing the time consuming Non-Linear Programming computations.

INTRODUCTION

In recent years it is well known that many countries are seriously interested in systematic knowledge and maintainance of their architectural tradition. A basic tool is the registration and survey of architectural monuments, as it is often emphasized in related meetings, symposia etc. *(e.g. ICOMOS, 1981)*.

This survey must include:

- Knowledge of the geometrical characteristics of the monument at a specific time
- Uniform accuracy
- The most possible metric and quality information
- Easy to approach Monument Intergrated Systems

All the above characteristics lead to the method of architectural photogrammetry *(Dallas, 1980)* which can fully satisfy all user needs, economically and quickly.

The method is devided in three main parts:

- Photo-coverage of the monument
- Measurements and observations on these photos
- Production of the final product (plan, section, thematic map, archive, orthophotography etc.)

The first two parts materialize the <u>photogrammetric network</u> as it is usually called in photogrammetry. The accuracy of the final product depends on the design of this net *(Zinndorf, 1986)*.

In this paper the optimization of the design of a two-photo photogrammetric network will be examined. This kind of the network, known also as stereo-pair, is often used in architectural surveys *(ICOMOS, 1981)*. The photos are assumed <u>metric</u>.

THE THEORETICAL ACCURACY OF THE STEREO-PAIR

Mathematic Model

The parameters of the mathematic model of the stereo-pair can be devided in two categories:

a. Those that define the geometric model, i.e.
   - The elements of photo exterior orientation. These consist of the camera station coordinates and cameras orientation angles.
   - The coordinates of the check points on the object

b. Those that define the stohastic model, i.e.
   - Random observation errors
   - Systematic errors caused by the degree of reliability of the mathematical model used to adjust the observations.

All the above information must be known in advance, in order to compute the accuracy of the survey. The difference between (theoretic) accuracy which depends on the observations' analysis and precision (or reliability) which depends on the closeness of observation to relity, must be reminded here *(see also Hottier, 1976).*

The mathematic model which is often used in photogrammetric networks is the collinearity equations *(Manual of Photogrammetry, 1984, p.88).* Every check point gives 4 such equations and the linearized system of observation equations can be written as

$$\begin{bmatrix} \dot{\mathbf{A}} & \ddot{\mathbf{A}} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \end{bmatrix} + \mathbf{v} = \mathbf{b} \qquad\qquad \mathbf{A}\,\mathbf{x} + \mathbf{v} = \mathbf{b} \qquad\qquad (1)$$

where "$\dot{}$" denotes parameters of exterior orientation and "$\ddot{}$" the check points coordinates and

$\mathbf{x}$: vector of parameters
$\mathbf{v}$: vector of photo coordinates corrections
$\mathbf{b}$: vector of differences between observed and approximate photo-coordinates

The solution is given by $\mathbf{N}\hat{\mathbf{x}} = \mathbf{A}^T\mathbf{P}\mathbf{b}$ (2)

where $\mathbf{N}$ is the normal equations matrix

$$\mathbf{N} = \begin{bmatrix} \dot{\mathbf{A}}^T \\ \ddot{\mathbf{A}}^T \end{bmatrix} \mathbf{P} \begin{bmatrix} \dot{\mathbf{A}} & \ddot{\mathbf{A}} \end{bmatrix} = \left[\begin{array}{c|c} \dot{\mathbf{A}}^T\mathbf{P}\,\dot{\mathbf{A}} & \dot{\mathbf{A}}^T\mathbf{P}\,\ddot{\mathbf{A}} \\ \hline \ddot{\mathbf{A}}^T\mathbf{P}\,\dot{\mathbf{A}} & \ddot{\mathbf{A}}^T\mathbf{P}\,\ddot{\mathbf{A}} \end{array}\right] \qquad (3)$$

and $\mathbf{P}$ is the weight matrix of the observatios. For check points accuracy analysis, the covariance matrix $\mathbf{N}^{-1}$ or $\mathbf{C}_{\hat{\mathbf{x}}}$ must be computed.

Partial inner constraints

The matrix $\mathbf{N}$ has a rank defect d=m-r=7 *(Meissl, 1969)*, where m is the number of parameters and r is the rank of $\mathbf{N}$, because the observations on the photos give no information for the reference ground coordinate system. So, the matrix $\mathbf{N}^{-1}$ cannot be computed unless 7 parameters are held constant, i.e.

$$\mathbf{Cx} = \mathbf{z} \tag{4}$$

where $\mathbf{C}$ is a known $d_{\times}m$ full rank (d) matrix and $\mathbf{z}$ a known vector. These constraints are also called minimum, because they are 7. It can be proved *(Meissl, 1965)* that the best choise of matrix $\mathbf{C}$ is not by choosing 7 parameters constant bat by establishing 7 relations between parameters, so that

$$\mathbf{Ex} = 0 \tag{5}$$

where $\mathbf{E}$ is the $d_{\times}m$ matrix of these relations. The method is known as "inner constraints" and has two basic properties: It minimizes the trace of $\mathbf{C}_{\hat{\mathbf{x}}}$ and the corrections of parameters, i.e.

$$\mathrm{tr}(\mathbf{C}_{\hat{\mathbf{x}}}) = \sum_{i=1}^{n} \sigma^2_{x_i} = \min \quad \& \quad \mathrm{tr}\left\{\begin{bmatrix} \hat{\mathbf{x}}^T & \hat{\ddot{\mathbf{x}}}^T \end{bmatrix} \begin{bmatrix} \hat{\mathbf{x}} \\ \hat{\ddot{\mathbf{x}}} \end{bmatrix}\right\} = \min \tag{6}$$

In our case matrix $\mathbf{E}$ is *(Dermanis, 1991)*

$$\begin{array}{ccccccccccccc} & \delta\kappa_i & \delta\phi_i & \delta\omega_i & \delta X_{oi} & \delta Y_{oi} & \delta Z_{oi} & \lrcorner & \delta X_j & \delta Y_j & \delta Z_j & \end{array}$$

$$\mathbf{E} = \begin{bmatrix} .. & 0 & 0 & 0 & 1 & 0 & 0 & ... & 1 & 0 & 0 & .. \\ .. & 0 & 0 & 0 & 0 & 1 & 0 & ... & 0 & 1 & 0 & .. \\ .. & 0 & 0 & 0 & 0 & 0 & 1 & ... & 0 & 0 & 1 & .. \\ .. & 0 & 0 & -1 & 0 & -Z_{oi} & Y_{oi} & ... & 0 & -Z_j & Y_j & .. \\ .. & \dfrac{\sin\omega_i}{\cos\phi_i} & -\cos\omega_i & -\sin\omega_i \tan\phi_i & Z_{oj} & 0 & -X_{oi} & ... & Z_j & 0 & -X_j & .. \\ .. & \dfrac{-\cos\omega_i}{\cos\phi_i} & -\sin\omega_i & \cos\omega_i \tan\phi_i & -Y_{oi} & X_{oi} & 0 & ... & -Y_j & X_j & 0 & .. \\ .. & 0 & 0 & 0 & X_{oi} & Y_{oi} & Z_{oi} & ... & X_j & Y_j & Z_j & .. \end{bmatrix}$$

elements of exterior orientation of photo i

check points' ncoords.m.

(7)

Using this matrix, the covariance matrix of the parameters is

$$\mathbf{C}_{\mathbf{x}E} = \mathbf{N}^{+} = (\mathbf{N} + \mathbf{E}^T\mathbf{E})^{-1} - \mathbf{E}^T(\mathbf{E}\mathbf{E}^T)^{-2}\mathbf{E}\mathbf{E} \tag{8}$$

where $\mathbf{N}^{+}$ is the pseudo-inverse of $\mathbf{N}$. An alternative form for (8) which is computationally easier can be obtained by similarity transformation of a minimum constrained solution

$$\mathbf{C}_{\mathbf{x}E} = \mathbf{S}\,\mathbf{C}_{\hat{\mathbf{x}}}\,\mathbf{S}^T \tag{9}$$

where $\mathbf{S}$ is the tranformation matrix

$$\mathbf{S} = \mathbf{I} - \mathbf{E}^T(\mathbf{E}\,\mathbf{E}^T)^{-1}\mathbf{E} \tag{10}$$

and $\mathbf{C}_{\hat{\mathbf{x}}}$ the covariance matrix of a minimum constrained solution. When we are interested in a subset of the network's parameters, $\mathbf{S}$ can be accordingly adjusted:

$$S_C = I - E^T(C\,E^T)^{-1}C \qquad (11)$$

where $C = [\ 0 \quad E\ ]$ for better check points accuracy, or $C = [\ \dot{E} \quad 0\ ]$ for better exterior orientation parameters accuracy.

The method of inner constraints can be easily applied in photogrammetric projects, as they are usually not referred to a specific ground coordinate reference system (as is often the case in geodesy), which can be independant.

In the case of monuments facades surveys, adjustment is usually devided in two stages:

- In the first, selecting a few points (e.g. 6) which cover all the facade and which are measured with topographic methods. These are the control points used for the determination of the 12 parameters of exterior orientation, $\dot{x}$.
- The measuring of any point of the object and computing its ground coordinates using the already computed 12 ext.orientation parameters.

Mathematically, this means that in the first stage apartial inner constraints adjustmentwhich minimizes the norm $[\dot{x}^T\dot{x}]$ can be used and then by using the parameters $\dot{x}$ and their covariance matrix, the covariance matrix of every measured check point can be computed. The covariance matrix of parameters $\dot{x}$ is *(Dimanidis, 1991)*

$$C_{\dot{x}C} = (I - \dot{E}^T(\dot{E}\,\dot{E}^T)^{-1}\dot{E})\ C_{\dot{x}}\ (I - \dot{E}^T(\dot{E}\,\dot{E}^T)^{-1}\dot{E})^T \qquad (12)$$

where $C_{\dot{x}}$ is the covariance matrix of $\dot{x}$ from a minimum constraints adjustment. The covariance matrix we finally need is

$$C_{\hat{\ddot{x}}} = N^{-1}\bar{N}\ C_{\dot{x}C}\bar{N}^T\ N^{-1} + N^{-1} \qquad (13)$$

STEREO-PAIR DESIGN OPTIMIZATION

The general case

The optimization problem which will be examined here is to attain the highest possible accuracy for points on a facade by changing the design of the stereo-pair, i.e. the parameters $\dot{x}$. The problem is known as first order design optimization *(Schmitt, 1985)*.

The specific criterion based on $C_{\hat{\ddot{x}}}$ which was used, is the sum of the volumes e_j of error ellipsoids of every check point on the facade is minimum:

$$\sum_{j=1}^{n}\left\{Vol(e_j)\right\} = min \qquad (14)$$

This criterion is known also in Statistics as D criterion and it is relative to the determinant of $C_{\hat{\ddot{x}}}$ and to the product of its eigenvalues.

Another critical point for the optimization procedure are the limits between which the parameters $\dot{x}$ can vary. These limits are defined by the

- Topography of the surroundings

- Coverage of the object in both photos
- Swing and scale limits of the photogrammetric analog instument (if used)

All these limitations can form equalities and inequalities *(Dimanidis, 1991)* which also contain the optimization parameters $\mathbf{x}$. These can be generally expressed as

$$h_i(\mathbf{x}) = 0 \ , \ i=1..m \quad \text{and} \quad g_i(\mathbf{x}) \geq 0 \ , \ i=m+1..p \qquad (15)$$

The problem of finding $\mathbf{x}$, as defined in (14) & (15), is a well known Non-Linear Programming (NLP) problem and can be solved with numerical methods *(Himmelblau, 1972, Chap.8)*.

The case of near-flat facades covered by symmetrical photo-pair.

In this case assuming that the object of the survey is near flat and in order to increase accuracy and stereoscopic viewing, some symmetries in photo-pair design can be established:

- Cameras at the same height
- Cameras at the same distance from the facade
- Same camera convergence and tilt

Swing round the projection axis (κ) can also be kept 0, as it does not influence object accuracy *(Zinndorf, 1986, p.52)*.

The optimization parameters are finally reduced to five, namely distance between the two cameras, i.e. base B, distance from facade D, camera height Y and camera convergence ϕ and tilt ω. All the above assumptions are valid. Exceptions for the established symmetries are due to surroundings limitations, as it is shown in the example given in the next paragraph.

When these limitations do not exist and the symmetric photo-pair is assumed, it can be proved that if the 5 optimum parameters B,D,Y,ϕ,ω for a specific facade with length L are computed, optimum ratios B/D and D/L will remain the same for every facade with length L'$\neq$L when ϕ and ω remain the same, but also when the length to height ratio remains the same *(Dimanidis, 1991)*.

$$\frac{B}{D} = \frac{B'}{D'} \quad \& \quad \frac{D}{L} = \frac{D'}{L'} \qquad (16)$$

For generalization purposes, the height of the facade can also be assumed variable, and in that case onother two ratio equalities are valid, namely

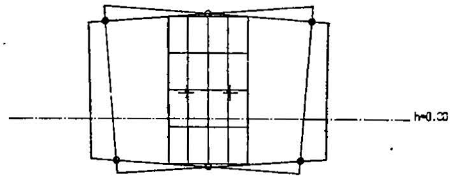
$$\frac{D}{Hup} = \frac{D'}{Hup'} \quad \& \quad \frac{D}{Hdown} = \frac{D'}{Hdown'} \qquad (17)$$

where Hup and Hdown are height differences of up and down limit of simulation facade from camera height.

In order to examine the accuracy of this constant length L and variable height simulation facade for any symmetrical photo-pair design configuration, check points must cover all this overlap "zone" of length L. One usual way of doing this is by assuming a grid over the facade and computing the optimization criterion (14) for every grid node. This technique is shown in

Fig. 1, where the two overlap photos of a stereo-pair and the "zone" of constant length 5m are drawn. The step of the grid must be such, until average value of criterion for all points doesnot significantly change by further increase of the step value.

In Table 1, 10 different symmetrical photo-pair configurations are presented, and the percent of the difference of mean error ellipsoids axis value for every grid density from the same value for max density (100 nodes for this example) to this value of max grid density.



θ/ω : WILD-P30, B = 10m, φ = 15g, ω = 7g, D = 15m, L = 5m, H = 9.4m

Fig.1 Grid on Constant length zone

B	D	φ	ω	number of check poins								
				4	9	16	25	36	49	64	81	100
13	6	42	14	8.9%	4.0%	2.3%	1.5%	0.9%	0.6%	0.3%	0.1%	0.0%
14	6	40	7	8.6%	3.9%	2.2%	1.4%	0.9%	0.6%	0.3%	0.1%	0.0%
19	9	44	21	4.9%	2.2%	1.3%	0.8%	0.5%	0.3%	0.2%	0.1%	0.0%
20	10	43	7	4.7%	2.1%	1.2%	0.8%	0.5%	0.3%	0.2%	0.1%	0.0%
20	12	26	0	4.3%	1.9%	1.1%	0.7%	0.4%	0.3%	0.2%	0.1%	0.0%
29	16	46	14	2.9%	1.3%	0.7%	0.5%	0.3%	0.2%	0.1%	0.0%	0.0%
34	26	24	0	1.9%	0.8%	0.5%	0.3%	0.2%	0.1%	0.1%	0.0%	0.0%
35	20	45	7	2.3%	1.0%	0.6%	0.4%	0.2%	0.1%	0.1%	0.0%	0.0%
36	20	45	21	2.7%	1.2%	0.7%	0.4%	0.3%	0.2%	0.1%	0.0%	0.0%
47	28	44	14	2.1%	0.9%	0.5%	0.3%	0.2%	0.1%	0.1%	0.0%	0.0%
average				4.3%	1.9%	1.1%	0.7%	0.4%	0.3%	0.2%	0.1%	0.0%

Table 1. Grid density dependency on value of accuracy criterion

According to this table, if 25 check points (grid nodes) are chosen, the average ellipsoid axis difference is under 1% of the 100 points average ellipsoid axis, which is a sutisfying value.

Optimum parameters can now be computed for various photo-pair design configurations and same facade length L, and a system of *k* linear equations be created:

$$\mathbf{A}\mathbf{x} + \mathbf{v} = \mathbf{b} \tag{18}$$

The equations can be polynomials so $\mathbf{x}=[\ a_1\ a_2 \ldots a_p\ ]^T$ where (p-1) is the polynomial degree, and

$$\mathbf{A} = \begin{bmatrix} x_1^{p-1} & x_1^{p-2} & \cdots\cdots & x_1 & 1 \\ x_2^{p-1} & x_2^{p-2} & \cdots\cdots & x_2 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ x_k^{p-1} & x_k^{p-2} & \cdots\cdots & x_k & 1 \end{bmatrix} \tag{19}$$

where x is ϕ, D/L, D/Hup, D/Hdown successively and $\mathbf{b} = [\ b_1\ b_2 \ldots b_k\ ]^T$ where b is B/D, ϕ, ϕ and ϕ correspondingly. Solution is given by

$$\hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}\mathbf{b} \tag{20}$$

The organization of the computations can be as follows:

1. Choose a camera
2. Choose a facade length
3. Choose a camera height
4. Choose a tilt angle
5. Optimize stereo-pairs for camera distances from facade between an upper and a lower limit (e.g. 50 - 4m).

For different tilt angles repeat steps 4,5. An example of polynomials coefficients polynomials is given in Table 2. Such polynomials can be obtained for any metric camera. To use them, compute first D/L and then from polynomial $f(D/L)=\phi$ with tilt ω, compute ϕ. With ϕ D/Hup and D/Hdown can be computed. If Hup or Hdown is not valid, try another tilt w' and compute parameters with an interpolation method between two polynomials for tilt $>\omega'$ and tilt $<\omega'$ *(e.g. the Aitken method, see F.Sheid, 1968, pp.54-55).*

D/L ▷ ϕ

ω	0.0	7.0	14.0	21.0
ϕ	19.530 - 24.630	31.936 - 45.600	33.914 - 46.000	35.785 - 46.000
1	2.79414845D+00	-3.60934973D+03	-1.61928250D+03	-9.54172069D+02
2	2.03330040D+00	7.15867087D+03	3.01922978D+03	1.82925881D+03
3	1.55740643D+00	-5.60553310D+03	-2.17761965D+03	-1.32886098D+03
4	-3.20430517D-01	2.19174098D+03	7.80615592D+02	4.79296368D+02
5	1.81782832D-02	-4.27654877D+02	-1.39045646D+02	-8.58265374D+01
6		3.32976245D+01	9.84259098D+00	6.10229678D+00

B/D ▷ ϕ

ω	0.0	7.0	14.0	21.0
ϕ	19.530 - 56.918	31.936 - 56.000	33.914 - 56.604	35.785 - 44.706
1	7.24124670D+01	7.27514326D+01	7.32018676D+01	7.66351417D+01
2	-3.95191068D+01	-3.89289412D+01	-3.90816641D+01	-5.55251885D+01
3	-9.98126841D+00	-6.74695230D+00	-4.64073753D+00	3.15018082D+01
4	4.22261183D+01	3.33042436D+01	3.02368374D+01	-1.01995504D+01
5	-3.71964164D+01	-2.57664549D+01	-2.35908117D+01	1.39203942D+00
6	1.71733424D+01	8.89908594D+00	8.11533594D+00	
7	-4.50860578D+00	-1.19919649D+00	-1.08478582D+00	
8	6.35791019D-01			
9	-3.74064655D-02			

Table 1. Optimum polynomial coefficients for symmetrical photo-pair

Optimization example

The method presented applied in the survey of a NE part of the byzantine church of "Agios Dimitrios" in Thessaloniki, Greece. The specific part is surrounded (see Fig.2) by other parts of the monument so it offers an interesting application of the method.

Two photo pairs were taken: An optimum (**03-04**)and a near-"normal case" one (**01-02**) in order to realize the significance of the optimization

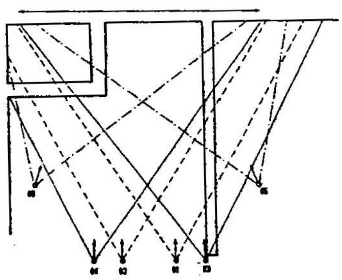


Fig.2 Photo-pairs for "Ag.Dimitrios"

procedure. Photo-pair **05-06** shown in Fig .2 is the optimum with no limitations, taken from polynomials. The facade was covered with 26 points (6 control and 20 check points) which were measured with a high accuracy 3-d topographic network. The parameters of the photo-pairs were:

01-02 : B=3.19m, D=14.32m, $\phi_1=-\phi_2=1^{grad}$, $\omega_1=\omega_2=0$

(near normal case)

03-04 : B=6.88m, D=14.32m, $\phi_1=-\phi_2=4.96^{grad}$, $\omega_1=\omega_2=0$

(optimum case)

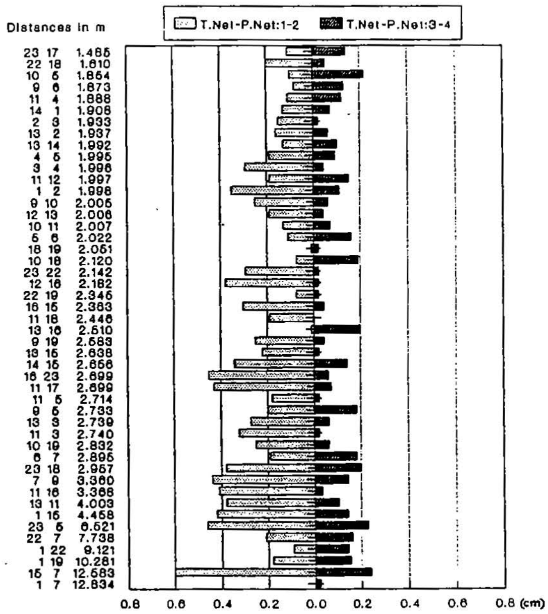


Fig.3 Comparison of two distances series

The 26 points were measured on the photots and the adjustment procedure was carried out using a bundle adjustment program. A criterion used to compare the two photo-pairs was the distances' differences between check points computed from the two solutions and those computed from the topographic network, which are shown in Fig.3. It can be easily seen that accuracy of 03-04 is almost twice as better from the 01-02 photo pair.

Conclusions

It has been shown how interesting and useful photo-pair design optimization can be for surveys of architectural monuments. The method presented needs no extra photos, just a better design of the photo-pair. In case of no surrounding limitations, optimization polynomials can be used, which by-pass the time-consuming general optimization method and so the optimization parameters can be computed on site.

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