# AN ENHANCEMENT OF THE DLT METHOD. FOR ARCHITECTURAL PHOTOGRAMMETRY APPLICATIONS

### D. Th. Panagiotidis

# ABSTRACT

An enhancement of the DLT method is presented. The enhanced DLT model comprises of the 11 DLT parameters, plus one for radial distortion, plus a polynomial of 10 additional parameters. All the unknown parameters for the two photos are incorporated in one Least Squares Solution. The additional parameters are checked to see if they are correlated or insignificant and if so they are discarded from the solution.

The method is tested with real data on two test fields and specific architectural monuments. A wide variety of non metric cameras covering an extended range of price, plus one metric camera are used.

Finally an economic data reduction system is proposed. This comprises of a non metric camera, the software described above, a microcomputer, a size A3 digitizer and a size A3 plotter.

#### INTRODUCTION

The DLT (Direct Linear Transformation) method is an analytical self calibration method that was developed to allow the use of non metric cameras in close range photogrammetry applications. (Abdel-Aziz & Karara, 1974). The DLT method makes no use of fiducials and is well accepted by photogrammetrists.

The success of the method depends on how well the distortions caused by non metric cameras are compensated for. The enhanced DLT model provides an improvement to solutions proposed until today. Even though the use of non metric cameras reduces the cost most applications of the method still incorporate the use of highly expensive instruments such as stereocomparators. It will be shown that by using a digitizer there is no significant loss of accuracy, while the cost can be considerably reduced.

# THE MATHEMATICAL MODEL OF THE DLT METHOD

The DLT method allows the Direct Linear Transformation from picture coordinates to ground coordinates, bypassing the intermediate stage of trans forming the picture coordinates from the comparator system to picture system. The DLT equations are the following:

$$x = \frac{L_{1}X + L_{2}Y + L_{3}Z + L_{4}}{L_{9}X + L_{10}Y + L_{11}Z + 1} \qquad y = \frac{L_{5}X + L_{6}Y + L_{7}Z + L_{8}}{L_{9}X + L_{10}Y + L_{11}Z + 1}$$

The 9 parameters of interior (c, x<sub>0</sub>, y<sub>0</sub>) and exterior ( $\omega \phi \kappa X_0 Y_0 Z_c$  orientation are interpreted through the 11 parameters Li.

A linearization of the DLT equations leads us to the following equations, regarding the observation of a single point i in picture j (Dermanis 1989):

$$\mathbf{b}_{ij} = \mathbf{A}_{ji} \mathbf{x}_{j} + \mathbf{A}_{ji} \mathbf{x}_{i} + \mathbf{v}_{ji}$$

where  $\mathbf{x}_{i}$  is the vector of unknown parameters L, and  $\mathbf{x}_{i}$  the vector of

ground coordinates. More specifically:

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$$\begin{aligned} \mathbf{\dot{x}_{j}}^{=} & \left[ \delta L_{1} \ \delta L_{2} \ \delta L_{3} \ \delta L_{4} \ \delta L_{5} \ \delta L_{6} \ \delta L_{7} \ \delta L_{8} \ \delta L_{9} \ \delta L_{10} \ \delta L_{11} \right]_{j}^{T} \\ \mathbf{\ddot{x}_{i}}^{=} & \left[ \ \delta X \ \delta Y \ \delta Z \ \right]_{1}^{T} \\ \mathbf{v}_{j1} & \left[ \ \mathbf{v}_{x} \ \mathbf{v}_{y} \ \right]_{j1}^{T} \\ \mathbf{b}_{j1}^{=} & \left[ \ \mathbf{x} - \mathbf{x}^{0} \ \mathbf{y} - \mathbf{y}^{0} \ \right]_{11}^{T} \end{aligned}$$

The design matrix is formed from the following submatrices which contain the partial derivatives of the DLT equations to the unknown parameters Li and the ground coordinates:

$$\mathbf{A}_{j1} = \begin{bmatrix} \frac{\partial \mathbf{x}}{\partial L_1} & \frac{\partial \mathbf{x}}{\partial L_2} & \cdots & \frac{\partial \mathbf{x}}{\partial L_{11}} \\ \frac{\partial \mathbf{y}}{\partial L_1} & \frac{\partial \mathbf{y}}{\partial L_2} & \cdots & \frac{\partial \mathbf{y}}{\partial L_{11}} \end{bmatrix}_{j1} \qquad \mathbf{A}_{j1} = \begin{bmatrix} \frac{\partial \mathbf{x}}{\partial X} & \frac{\partial \mathbf{x}}{\partial Y} & \cdots & \frac{\partial \mathbf{x}}{\partial Z} \\ \frac{\partial \mathbf{y}}{\partial X} & \frac{\partial \mathbf{y}}{\partial Y} & \cdots & \frac{\partial \mathbf{y}}{\partial Z} \end{bmatrix}_{j1}$$

For each pair of observations of photo coordinates of a point i in picture we assume the following equations:

$$\begin{bmatrix} \dot{\mathbf{N}}_{j1} & \overline{\mathbf{N}}_{j1} \\ \overline{\mathbf{N}}^{\mathrm{T}} & \mathbf{N}_{j1} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{j} \\ \mathbf{\ddot{x}}_{i} \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{j1} \\ \mathbf{\ddot{u}}_{j1} \end{bmatrix}$$
where:  
$$\dot{\mathbf{N}}_{j1} = \mathbf{A}_{j1}^{\mathrm{T}} \mathbf{P}_{j1} \mathbf{A}_{j1} \qquad \mathbf{a}_{j1} = \mathbf{A}_{j1}^{\mathrm{T}} \mathbf{P}_{j1} \mathbf{b}_{j1}$$
$$\mathbf{N}_{j1} = \mathbf{A}_{j1}^{\mathrm{T}} \mathbf{P}_{j1} \mathbf{\ddot{A}}_{j1} \qquad \mathbf{\ddot{u}}_{j1} = \mathbf{A}_{j1}^{\mathrm{T}} \mathbf{P}_{j1} \mathbf{b}_{j1}$$
$$\overline{\mathbf{N}}_{j1} = \mathbf{A}_{j1}^{\mathrm{T}} \mathbf{P}_{j1} \mathbf{\ddot{A}}_{j1} \qquad \text{and}$$
$$\mathbf{P}_{j1} = \mathbf{C}_{j1}^{-1} = \begin{bmatrix} \sigma_{\mathbf{x}}^{2} & \sigma_{\mathbf{x}y} \\ \sigma_{\mathbf{yx}} & \sigma_{\mathbf{y}}^{2} \end{bmatrix}_{j1}^{-1}$$

Since the picture coordinates are measured with the same accuracy and are not correlated we assume that  $\sigma_x^2 = \sigma_y^2 = \sigma_0^2$  and the weight matrix  $\mathbf{P}_{ji}$  is replaced by  $1/\sigma_o^2 \times \mathbf{I}$ .

The systematic errors were treated as follows: a) Radial lens distortion. A third degree polynomial was chosen.  $\delta_{1} = K_{1} r^{3}$  and the corrections for the x and y picture coordinates are

$$\Delta x_{rad} = x' K_1 r^2 \qquad \Delta y_{rad} = y' K_1 r^2$$

b) Other distortions of lens and film. One of the models suggested by Ebner was chosen, containing 12 additional parameters.

where  $ba=b^2/3$ 

$$\Delta x_{ad} = b_1 x + b_2 y - b_3 (2x^2 - 4ba) + b_4 xy + b_5 (y^2 - 2ba) + b_7 x (y^2 - 2ba) + + b_9 y ((x^2 - 2ba) + b_{11} (x^2 - 2ba) (y^2 - 2ba) + x_0$$
  
$$\Delta y_{ad} = -b_1 y + b_2 x + b_3 xy - b_4 (2y^2 - 4ba) + b_6 (x^2 - 2ba) + b_8 y (x^2 - 2ba) + + b_1 x ((y^2 - 2ba) + b_{12} (x^2 - 2ba) (y^2 - 2ba) + y_0$$
 where  $ba = b^2/3$ 

The enhanced suggested DLT model is the following:

$$x + \Delta x_{rad} + \Delta x_{ad} = \frac{L_1 X + L_2 Y + L_3 Z + L_4}{L_9 X + L_{10} Y + L_{11} Z + 1}$$
$$x + \Delta x_{rad} + \Delta x_{ad} = \frac{L_5 X + L_6 Y + L_7 Z + L_8}{L_9 X + L_{10} Y + L_{11} Z + 1}$$

The solution with the observation equations method becomes:

$$\mathbf{b}_{ij} = \begin{bmatrix} \mathbf{A}_{ji} & \mathbf{D}_{ji} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{j} \\ \mathbf{y}_{j} \end{bmatrix} + \mathbf{A}_{ji} \mathbf{x}_{i} + \mathbf{v}_{ji}$$

Matrix **D** contains the partial derivatives to all the distortion parameters, while the parameters themselves are contained in vector  $\mathbf{y}$ .

# SELECTING A POLYNOMIAL OF ADDITION PARAMETERS

The selection of a polynomial of the suggested form for the compensation of systematic errors and generally of any form introduces an empirical solution to a complicated problem such as self calibration.

Usually one would choose a polynomial with many parameters aiming to cover all kinds of systematic errors. This automatically introduces the need of examining the determinability of the unknown parameters. There is a possibility that some parameters be overdetermined by others, while others may be correlated, resulting in a weak solution, or insignificant.

On the other side it is almost impossible to compensate for the distor tions of any non metric camera by introducing a general polynomial of addi tional parameters, since the use of another camera or even the same one, but with different focusing, and the lack of proper film flattening, will make necessary the introduction of new distortion parameters.

Therefore one faces the problem of selecting only the necessary parameters for the compensation of distortions. (Panagiotidis, 1991). In this project the following procedure was applied:

The strongly correlated parameters were detected and discarded from the solution. The selection took place after the inverse of matrix N was computed and after the correlation coefficients were calculated. A threshold was chosen, above which a parameter was considered correlatd. Gruen (1985) suggests the value of 0.9 for this threshold. In this project we also tried 0.75 and 0.65. A t-test followed to find out whether the contribution of a considered parameter in the overall compensation was significant.

# APPLICATION OF THE ENHANCED DLT METHOD

The project followed these stages:

1) Establishment of test fields, 2) Use of an extended number of non metric (and metric) cameras, 3) Software development, 4) Consideration of the effect of various factors in accuracy, 5) Suggestion for the development of an analytical system.

The test field allowed the positioning of targets on two planes 4 meters apart. Plane No 1 contained 16 targets and No 2 27. Targets of 5.5 cm diameter were used with 1 mm diameter centre (Fig. 1).

Ground control was provided by a triangulation network. From 4 stations all the directions and 5 distances were measured. The network, with overall 47 points and 367 observation equations, was adjusted as a three-dimensional one with program ICONA3 (Rossikopoulos, 1986). Two stations were fixed in X and Y and one in Z. The a-posteriori value of variance was 1.06 (a-priori=1) and the semiaxes of error ellipsoids of targets varied from 0.4 mm to 0.8 mm for X and Y and from 0.1 mm to 0.2 mm for Z.

The final values for the ground coordinates from the adjustment were cosidered true and were used for the evaluation of the DLT results.

A second test field was also set up to test non metric cameras with the DLT method. 14 targets (7 on each plane) were used. Ground control was also provided by a triangulation network and a three - dimensional least squares adjustment.

CAMERA	Pr.Dist mm	BASE m	DIST. m	B /Y	CONVERG. grad	CODE
Pentax Me S	50	1.0	7.0	1/7.0	2	MES502
Pentax Me S	50	1.4	7.0	1/5.0	3	MES501
Pentax Me S	50	2.7	7.0	1/2.6	13	MES504
Pentax Me S	50	3.2	7.0	1/2.2	15	MES503
Pentax Me S	80	3.2	11.0	1/3.4	8	MES801
Pentax Pr A	50	1.0	7.0	1/7.0	1	PRA501
Pentax Pr A	50	1.4	7.0	1/5.0	5	PRA502
Pentax Pr A	28	0.8	4.0	1/5.0	_1	PRA282
Pentax Pr A	28	1.4	4.0	1/2.9	12	PRA281
Zenit E	52	2.5	8.5	1/3.4	8	ZEN501
Zenit E	52	3.3	8.5	1/2.6	11	ZEN502
Hasselblad	80	2.5	6.3	1/2.5	12	HAS801
Kodak	40	1.2	7.0	1/5.8	3	KOD422
Kodak	40	3.2	7.0	1/2.2	13	KOD421
P-32	64	1.0	4.0	1/4.0	13	P32641
					×	1

Table 1. The settings tested on test field No1.

Aim of the project was the testing of an extended variety of amateur cameras, as well as the behaviour of the method in extreme or difficult conditions. Former publications (Abdel-Aziz & Karara, 1974) had already

proved the fidelity of the method, but in rather favourable conditions considering the number and distribution of control points, base length and distance of photography.

	CAMERA	LENS	X	Y	z	conv grad	B/Y
	PENTAX ME Super	60	0.5	0.8	3.6	3	5.0
	PENTAX ME Super	50	1.1	1.1	6.1	2	7.0
	PENTAL ME Super	50	0.6	0.6	2.0	16	2.2
	PENTAX ME Super	60	0.5	0.5	3.2	13	2.6
	PENTAX ME Super	80	0.7	0.8	4.5	8	3.4
	PENTAX PROGRAM A	50	0.5	0.4	9.2	5	6.0
	2 60 m PENTAX PROGRAM A	50	0.7	0.6	4.1	1	7.0
	PENTAX PROGRAM A	28	3.0	2.4	19.9	12	2.9
	PENTAX PROGRAM A	28	6.3	2.6	24.8	1	6.0
	HASSELBLAD 600 C	80	0.4	0.2	1.3	12	2.5
• • • • • • •	ZENIT B	60	0.7	0.8	3.5	8	3.4
	ZENIT E	60	0.8	1.0	2.5	11	2.5
4.10. m	KODAK	42	4.8	4.1	22.3	3	5.8
• •	KODAK	42	1.6	1.6	8.3	13	2.2
	WILD P32	64	0.4	0.6	1.5	13	4.0
3.15 m	WILD P32	64	0.9	0.8	2:8	2	4.0

Fig.1 The test field

#### Table 1.General Results

On test field No 1 the cameras shown coded in table 1 were used. Photography took place between 11.00 and 14.00 in natural daylight. Agfapan 25 and 100 ASA was used in all cameras. Five shots were made with each camera-lens combination.

Negative scale was 1:167 for Kodak, 1:78 for Hasselblad, 1:64 for Wild P32 and varied from 1:135 to 1:147 for the rest. Next the negatives were placed on stereocomparator Wild STK-1, and the photocoordinates were measured stereoscopically in two sets. The standard deviation of the readings was found to be 2.8 microns (out of a sample of 50 repeated readings).

### RESULTS

Most applications of the DLT method apply redundant control. Abdel- Aziz & Karara, (1974) suggest redundant ground control and use 33 points as control points. All points are also used by Karara (1974). Fraser (1982) uses 57 control points, while in many publications this number is not mentioned.

To investigate the effect of the number of control points in accuracy, there were many solutions all with redundant control. 12 control points were used as the basic solution for all camera-lens combinations. All other possible solutions were also tested (i.e. with 6, 7, 8, 9, 10, 11 points, 16 and finally all points available).

In all cases the basic solution of 12 control points produced satisfactory results. For X and Y coordinates the R.M.S. of differences (from ground control) varied from 0.5 mm to 1.0 mm, while for Z (depth) from 1.5 mm to 6.5 mm, with the only exceptions of the cheapest Kodak camera and the wide angle lens Pentax 28 mm (See Table 2).

Comparing the results with distances measured on site the differences varied for the best cameras from 0 to 1 mm with standard deviation of 0.5 mm (sample of 28 distances) and from 1 to 5 mm with s.d. of 3.5 mm for the others.

One can also conclude that 9 control points is a critical point and represent the minimum requirementes. Also 12 control points are a safe and and economical solution.

The effect of the additional parameters was investigated also by means of solutions with redundant control. In the following charts on can compare

solutions (Table 3):

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a) with only radial lens distortion compensation, b) as case (a) plus 10 additional parameters, c) automatic selection of parameters with correlation check and t-test.



Table 3. The effect of the additional parameters

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The charts relate to three non metric cameras. The three bars' groups represent values for X, Y, Z coordinates and independent values for each plane of the test field are given. One can reach the following conclusions:

1) Compensation for only radial lens distortion seems to be enough for the correction of pic coordinates and the user could easily stop there.

2) In cases of good cameras (WIId, Hasselblad, Pentax) but also in cases of good settings, it is sometimes difficult to come to conclusions since the achieved accuracy can reach the limits of the ground control accuracy.

3) In all cases the addition and keeping of all additional parameters led to significant decrease of accuracy (comp. cases II with I, III, IV).

4) The results with both tests are of the same quality, if not better, to those with only radial distortion compensation (comp. case IV with I).

5) There are no residual systematic errors, while these appear if all additional parameters are incorporated.

6) The increase in accuracy from the radial-distortion-only solution to the one with both checks is of the 5-20% order for the X, Y plane and of 5% for Z (depth). Regarding length measurements on site this increase is 5-10% respectively.

# USING A DIGITIZER INSTEAD OF A COMPARATOR

In all cases the stereocomparator Wild STK-1 was used for the measurement of photo coordinates. The STK-1 is a very precise instrument, large in size and expensive. Therefore it was decided to try the method using a less precise (and less expensive) comparator. Since there was no such comparator available, the readings of the STK-1 were used to simulate two comparators with accuracy of reading 10 and 100 microns respectively. The input values for case MES503 were used (12 control points).

The results are given in the next table and one can see that accuracy in the order of 10 microns in the comparator reading is enough to produce satis factory results, while in extreme cases one can accept coordinates with accuracy in the order of about 100 microns.

sX mm	sY mm	sZ mm
0.6	0.6	2.0
1.0	1.1	6.0
8.0	8.2	52.3
	sX mm 0.6 1.0 8.0	sX sY   mm mm   0.6 0.6   1.0 1.1   8.0 8.2

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Achieved accuracy with real data (1 micron) and simulated data (10 and 100 microns)

Modern digitizers tend to reach these levels of accuracy (10-30 microns), having good chances to be included in instrumentation lists. Therefore after that simulated test, a series of adjustments followed using digitizers for the measurement of coordinates. Two digitizers were used: Houston Instruments Hipad plus 9018 and Hitachi.

Enlargement prints were made from the original negatives (ratio 4:1). The prints were made on the rectifier WILD E4, to avoid the introduction of further distortions.

CAMERA	Pentax	Pentax	Pentax	Wild P32	Zenit E
PICTURE	NEGATIVE	PRINT	PRINT	PRINT	PRINT
MEASUREMENT	Wild STK1	Hitachi	Houston I	Houston I	Houston I
No of SETS	2	2	2	3	3
CNTR. POINTS	12	12	12	13	13
ACCURACY XY (mm)	0.6 0.6 2.0	2.4 3.0 12.5	2.7 3.2 18.4	2.1 2.0 14.1	2.4 2.5 17.1

Comparative results of comparator and digitizers

The Hitachi digitizer claimed a resolution of 40 lines/mm. For the Houston Instruments Digipad 9018 this value was 100 lines/mm even though its standard deviation was found to be 40 lines/mm for the X and 110 for the Y axes (out of a 100-values sample). The photos used were: a) Prints of Pentax (MES503) on Wild E4 rectifier, b) Prints of Wild P-32 on Wild E4 rectifier c) Prints of Zenit on an amateur Krokus enlarger

The final ground coordinates differed, from those geodetically known, in less than 1 mm in X and Y, while in Z from 2.5 to 5 mm. The differences from lentgh measurements on site varied from 1 to 5 mm. Another case of using a digitizer was a photograph of a facade of Saint Demetrius church in Thessaloniki (Panagiotidis, 1991). Again the Houston Instruments digitizer was used. The ground coordinates of 21 target-points were calculated and the R.M.S. of differences was sX = 6 mm, sY = 6 mm, sZ = 15 mm. These values can be directly compared with values which resulted from comparator observations and equal distribution of control points in two planes.

#### GENERAL CONCLUSIONS

In conclusion it was proved that the method can be applied on architectural monuments, rich in detail. The method provides an alternative to analog methods, with significant reduce of instrumentation cost. Other advantages are less field work, minimum requirements in personell and the ability to produce practically infinite number of points and make additions whenever necessary. The cost of instrumentation for the set-up described is:

Comoro		100 000	
Camera		100.000	
Digitizer A3		250.000	
P/C AT or 386		400.000	
Plotter A3		300.000	
	total	1.050.000	drs

It is believed that the experience from this project will make evident to engineers that Analytical Photogrammetry methods can produce satisfactory results and be combined with topographic methods, using simple non metric cameras and affordable instrumentation

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